TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Description Logics

Exercise Sheet 12

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Exercise 44

For each of the following ExpTime games, determine whether Player 2 has a winning strategy:

a) $G_1 = (p \to q, \{p\}, \{q\}, \{p \mapsto 1, q \mapsto 0\})$,

b) $G_2 = (p \to q, \{q\}, \{p\}, \{p \mapsto 1, q \mapsto 0\})$,

c) $G_3 = (p_1 \wedge p_2 \leftrightarrow q, \{p_1, p_2\}, \{q\}, \{p_1 \mapsto 0, p_2 \mapsto 0, q \mapsto 1\})$,

d) $G_4 = (\phi, \Gamma_1, \Gamma_2, t_0)$ with
- $\phi = (p_1 \wedge p_2 \wedge p_3 \wedge \neg q) \vee (\neg p_1 \wedge \neg p_2 \wedge \neg p_3 \wedge q)$,
- $\Gamma_1 = \{p_1, p_2, p_3\}$,
- $\Gamma_2 = \{q\}$,
- $t_0(p_1) = t_0(p_2) = t_0(p_3) = t_0(q) = 0$.

Exercise 45

The description logic $\mathcal{S}$ extends $\mathcal{ALC}$ with *transitivity axioms* $\mathsf{trans}(r)$ for roles $r \in N_R$. The semantics are defined as follows: $\mathcal{I} \models \mathsf{trans}(r)$ if, and only if, $r^{\mathcal{I}}$ is transitive. Furthermore, an $\mathcal{S}$-knowledge base $(\mathcal{T}, \mathcal{A}, \mathcal{R})$ consists of an $\mathcal{ALC}$-knowledge base $(\mathcal{T}, \mathcal{A})$ and a *RBox* $\mathcal{R}$ of transitivity axioms. Prove the following claims:

a) $\mathsf{trans}(r)$ cannot be expressed in $\mathcal{ALC}$, i.e., $\mathcal{S}$ is more expressive than $\mathcal{ALC}$.

Hint: Show that the FOL-formula $\forall x.\forall y.\forall z.(R(x,y) \wedge R(y,z)) \to R(x,z)$ is not equivalent to a formula in the two-variable-fragment of FOL.

b) For an arbitrary TBox $\mathcal{T}$, the concept description $C_{\mathcal{T}}$ is defined as $\bigsqcap_{C \sqsubseteq D \in \mathcal{T}} \neg C \sqcup D$. Then $\mathcal{T}$ and $\{\top \sqsubseteq C_{\mathcal{T}}\}$ have the same models.

c) Let $\mathcal{K} = (\mathcal{T}, \mathcal{A}, \mathcal{R})$ be a $\mathcal{S}$-knowledge base such that w.l.o.g. $\mathcal{T}$ consists of a single GCI $\top \sqsubseteq C_{\mathcal{T}}$, and $C_{\mathcal{T}}$ is in NNF. Define the $\mathcal{ALC}$-knowledge base $\mathcal{K}^+ := (\mathcal{T}^+, \mathcal{A})$ where $\mathcal{T}^+ := \mathcal{T} \cup \{\forall r.C \sqsubseteq \forall r.\forall r.C \mid \mathsf{trans}(r) \in \mathcal{R}, \forall r.C \in \mathsf{Sub}(C_{\mathcal{T}})\}$.

Then $\mathcal{K}$ is consistent if, and only if, $\mathcal{K}^+$ is consistent.

Consequently, the tableaux algorithm for $\mathcal{ALC}$ can also be utilized for $\mathcal{S}$.

d) The problem of deciding consistency of a $\mathcal{S}$-knowledge base (with a general TBox) is ExpTime-complete.

Exercise 46

Let $f_1, \ldots, f_m$ and $g_1, \ldots, g_n$ be (not necessarily distinct) abstract features. A *feature agreement* is a concept of the form $(f_1 \circ \cdots \circ f_m) \downarrow (g_1 \circ \cdots \circ g_n)$ with the semantics:

$$((f_1 \circ \cdots \circ f_m) \downarrow (g_1 \circ \cdots \circ g_n))^{\mathcal{I}} := \{d \in \Delta^{\mathcal{I}} \mid f_m^{\mathcal{I}}(\cdots f_2^{\mathcal{I}}(f_1^{\mathcal{I}}(d))\cdots) = g_n^{\mathcal{I}}(\cdots g_2^{\mathcal{I}}(g_1^{\mathcal{I}}(d))\cdots)\}$$

Feature disagreements ($\uparrow$) are defined analogously. The description logic $\mathcal{ALCF}$ extends $\mathcal{ALC}$ with feature agreements and feature disagreements.

Show that satisfiability w.r.t. general TBoxes is undecidable for $\mathcal{ALCF}$.

Exercise 47

A (normalized) Horn-$\mathcal{FL}_0$-TBox may contain GCIs of the following forms (where $A, B, C \in N_C$, and $r \in N_R$):

$$A \sqsubseteq \bot, \quad \top \sqsubseteq A, \quad A \sqsubseteq B, \quad A \sqcap B \sqsubseteq C, \quad A \sqsubseteq \forall r.B.$$

Furthermore, (normalized) Horn-$\mathcal{FL}_0$-ABoxes consist of axioms $A(a)$ or $r(a, b)$ (where $A \in N_C$, $r \in N_R$, and $a, b \in N_I$). Prove that checking consistency of (normalized) Horn-$\mathcal{FL}_0$-knowledge bases can be done in polynomial time.

Hint: Satisfiability of propositional Horn-formulae is in P.