Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Description Logics

Exercise Sheet 13

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Exercise 48

Consider the concrete domain $\mathcal{N} := (\Delta^{\mathcal{N}}, \Phi^{\mathcal{N}})$ defined in Section 7.2 of the lecture. Show that $\mathcal{N}$ is admissible.

$$\Delta^{\mathcal{N}} := \mathbb{N}$$
$$\Phi^{\mathcal{N}} := \{=, =_0, +1, \top_{\mathcal{N}}\} \cup \{\text{negations of } =, =_0, +1, \top_{\mathcal{N}}\}$$

Exercise 49

Let $\mathcal{D}$ be a concrete domain and $\mathcal{ALC}(\mathcal{D})$ denote the extension of $\mathcal{ALC}$ with the concrete domain $\mathcal{D}$. Show the following statements:

a) If f is an abstract feature, then $\exists f.C$ is equivalent to $\exists f.\top \sqcap \forall f.C$.

b) If $\mathcal{D}$ contains only unary predicates, every $\mathcal{ALC}(\mathcal{D})$-concept can be 'emulated' by a corresponding $\mathcal{ALCN}$-concept.

Exercise 50

Consider the concrete domain $\mathcal{B}$ where $\Delta^{\mathcal{B}} := \{0, 1\}$, and the predicate set is given as

(i) $\Phi^{\mathcal{B}} := \{\bot_{\mathcal{B}}, \top_{\mathcal{B}}, =_0, =_1\}$,

(ii) $\Phi^{\mathcal{B}} := \{\phi(x_1, \dots, x_n) \mid n \in \mathbb{N}$ and ϕ is a propositional formula over the variables $x_1, \dots, x_n\}$ such that

$$(\phi(x_1, \dots, x_n))^{\mathcal{B}} := \{(v_1, \dots, v_n) \mid v_1, \dots, v_n \in \{0, 1\} \text{ and } \phi[x_1 \mapsto v_1, \dots, x_n \mapsto v_n] \equiv 1\}.$$

Prove the following claims:

a) $\mathcal{B}$ is admissible.

b) Satisfiability of $\mathcal{ALC}(\mathcal{B})$-concepts w.r.t. general TBoxes is ExpTime-complete.

Exercise 51

Let L be a language over the alphabet Σ, and consider the concrete domain

$$\mathcal{D}_L \coloneqq (\Sigma^*, \{\bot_{\mathcal{D}_L}, \top_{\mathcal{D}_L}, \in L, \notin L\}).$$

For each of the following cases, check whether $\mathcal{D}_L$ is admissible, and if concept satisfiability is decidable - both without a TBox and w.r.t. a general TBox.

a) $L \in \mathsf{P}$

b) $L \in \mathsf{PSpace}$

c) $L \in \mathsf{ExpTime}$

d) L is undecidable