TECHNISCHE UNIVERSITÄT DRESDEN

**Faculty of Computer Science** Institute of Theoretical Computer Science, Chair of Automata Theory

# Description Logics

## Exercise Sheet 14

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### Exercise 52

Prove or refute the following claim:

For a TBox $\mathcal{T}$ there can be exponentially many MinAs for a consequence $A \sqsubseteq_{\mathcal{T}} B$.

### Exercise 53

Let $\mathcal{L}_1$ and $\mathcal{L}_2$ be two DL-languages, and $C_1, \dots, C_n$ be $\mathcal{L}_1$-concepts. A $\mathcal{L}_2$-concept $C$ is called *$\mathcal{L}_2$-least common subsumer (lcs) of* $C_1, \dots, C_n$ if the following conditions are satisfied:

(1) $C_i \sqsubseteq C$ for all $i$, $1 \leq i \leq n$, and

(2) for all $\mathcal{L}_2$-concepts $D$ it holds that $C \sqsubseteq D$ if $C_i \sqsubseteq D$ for all $i$, $1 \leq i \leq n$.

This generalizes the standard notion of least common subsumers, since it allows different logics for $C_1, \dots, C_n$ and their lcs.[1]

Consider the following $\mathcal{FLE}$-concepts:

$$C_1 = \exists r.C \sqcap \exists r.D \sqcap \forall r.(A \sqcap B),$$
$$C_2 = \exists r.B \sqcap \exists r.D \sqcap \forall r.(A \sqcap D).$$

Find

a) the $\mathcal{EL}$-lcs,

b) the $\mathcal{FLE}$-lcs[2], and

c) the $\mathcal{ALC}$-lcs

of $C_1$ and $C_2$.

[1] It is easily verified that all $\mathcal{L}_2$-lcs of $\mathcal{L}_1$-concepts $C_1, \dots, C_n$ are unique up to equivalence, and hence we may speak of *the* lcs.

[2] $\mathcal{FLE}$ provides conjunction, existential restrictions, and value restrictions.

## Exercise 54

Let $\mathcal{K} = (\mathcal{A}, \mathcal{T})$ be a knowledge base, and $i_1, \dots, i_n$ be individuals occuring in $\mathcal{A}$.

A concept $C$ is called *most specific concept of* $i_1, \dots, i_n$ *w.r.t.* $\mathcal{K}$ if it satisfies the following statements:

(1) all individuals $i_1, \dots, i_n$ are instances of $C$ w.r.t. $\mathcal{K}$, and

(2) for all concept descriptions $D$ it holds that $C \sqsubseteq_{\mathcal{T}} D$ if all individuals $i_1, \dots, i_n$ are instances of $D$ w.r.t. $\mathcal{K}$.

Prove that the most specific concept of $i_1, \dots, i_n$ w.r.t. $\mathcal{K}$ is equivalent to the least common subsumer w.r.t. $\mathcal{T}$ of the most specific concepts of $i_1, \dots, i_n$ w.r.t. $\mathcal{K}$, i.e.

$$\mathsf{msc}_{\mathcal{K}}(i_1, \dots, i_n) \equiv \mathsf{lcs}_{\mathcal{T}}(\mathsf{msc}_{\mathcal{K}}(i_1), \dots, \mathsf{msc}_{\mathcal{K}}(i_n))$$