Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Description Logics

Winter Semester 2016

Exercise Sheet 2

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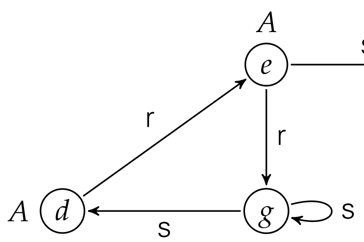
Exercise 2.5 Extend the mapping τ_x of $\mathcal{ALC}$-concept descriptions to first-order formulas given in the lecture to the description logic $\mathcal{ALCQ}$, which augments $\mathcal{ALC}$ with qualified number restrictions.

Exercise 2.6 Recall that the description logic $\mathcal{ALC}$ is equipped with the concept constructors negation ($\neg$), conjunction ($\sqcap$), disjunction ($\sqcup$), existential restriction ($\exists r.C$), and universal restriction ($\forall r.C$). Each subset of this set of constructors gives rise to a fragment of $\mathcal{ALC}$.

Identify all minimal fragments that are equivalent to $\mathcal{ALC}$ in the sense that for every $\mathcal{ALC}$-concept, there is an equivalent concept in the fragment. (Two concepts are equivalent iff they have the same extension in every interpretation.)

Exercise 2.7 Consider the (graphical representation of the) interpretation $\mathcal{I}$ with $\Delta^{\mathcal{I}} = \{d, e, f, g\}$:



For each of the following $\mathcal{ALCNI}$-concepts C, list all elements x of $\Delta^{\mathcal{I}}$ such that $x \in C^{\mathcal{I}}$:

(a) $A \sqcup B$

(b) $\exists s.\neg A$

(c) $\forall s.A$

(d) $(\geq 2\ s)$

(e) $\exists s.\exists s.\exists s.\exists s.A$

(f) $\forall s^{-1}.\exists s.\exists s.\exists s.A$

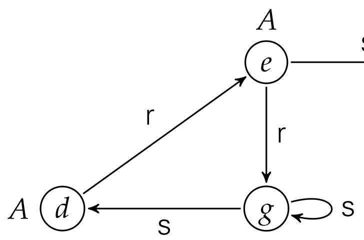
(g) $\neg\exists r.(\neg A \sqcap \neg B)$

(h) $\exists s.(A \sqcap \forall s.\neg B) \sqcap \neg\forall r.\exists r.(A \sqcup \neg A)$

Exercise 2.8 Consider the ABox

$$\mathcal{A} = \{A(d),\ A(e),\ A(f),\ B(f),\ r(d,e),\ r(e,g),\ s(e,f),\ s(g,g),\ s(g,d)\}$$

with the following graphical representation:



For each of the following $\mathcal{ALC}$-concepts C, list all individuals that are instances of C w.r.t. $\mathcal{A}$. Compare your results to Exercise 7.

(a) $A \sqcup B$

(b) $\exists s.\neg A$

(c) $\forall s.A$

(d) $\exists s.\exists s.\exists s.\exists s.A$

(e) $\neg\exists r.(\neg A \sqcap \neg B)$

(f) $\exists s.(A \sqcap \forall s.\neg B) \sqcap \neg\forall r.\exists r.(A \sqcup \neg A)$

Exercise 2.9 Revisit the procedure for expanding TBoxes given in the proof of Proposition 2.6 of the lecture. Prove that

(a) this procedure always terminates, and

(b) that it returns a TBox that is equivalent to its input.

Hint for proving termination: Count, for each concept name A, the number of concept names (directly or indirectly) used in the definition of A.