TECHNISCHE UNIVERSITÄT DRESDEN

**Faculty of Computer Science** Institute of Theoretical Computer Science, Chair of Automata Theory

# Description Logics

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## Exercise Sheet 3

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**Exercise 3.10** Consider the TBox

$$\mathcal{T} := \{\neg(A \sqcup B) \sqsubseteq \bot, \quad A \sqsubseteq \neg B \sqcap \exists r.B, \quad D \sqsubseteq \forall r.A, \quad B \sqsubseteq \neg A \sqcap \exists r.A\},$$

the ABox

$$\mathcal{A} := \{r(a,b),\ r(a,c),\ r(a,d),\ r(d,c),\ (B \sqcap \forall r.D)(a),\ E(b),\ (\neg A)(c),\ (\exists s.\neg D)(d)\},$$

and the knowledge base $\mathcal{K} := \langle \mathcal{T}, \mathcal{A} \rangle$. Check for

(a) the TBox $\mathcal{T}$,

(b) the ABox $\mathcal{A}$, and

(c) the knowledge base $\mathcal{K}$,

whether it has a model. If it has one, specify such a model. If it does not have a model, explain why.

**Exercise 3.11** Prove that existential restrictions are monotonic, i.e. show that

$$C \sqsubseteq_{\mathcal{T}} D \quad \Longrightarrow \quad \exists r.C \sqsubseteq_{\mathcal{T}} \exists r.D.$$

**Exercise 3.12** Prove the following result: Let $\mathcal{K} := \langle \mathcal{T}, \mathcal{A} \rangle$ be a knowledge base. If $a$ is an instance of $C$ w.r.t. $\mathcal{K}$ and $C \sqsubseteq_{\mathcal{T}} D$, then $a$ is an instance of $D$ w.r.t. $\mathcal{K}$.

**Exercise 3.13** Prove the following results. Let $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ be a knowledge base, $C$ an $\mathcal{ALC}$-concept, and $a$ an individual name.

(a) $\mathcal{K}$ is consistent iff $\tau(\mathcal{K})$ is consistent.

(b) $a$ is an instance of $C$ w.r.t. $\mathcal{K}$ iff $\tau(\mathcal{K}) \models \tau_x(C)(a)$.

**Exercise 3.14** Let $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ be a consistent knowledge base. We write $C \sqsubseteq_{\mathcal{K}} D$ if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ for every model $\mathcal{I}$ of $\mathcal{K}$. Prove that for all $\mathcal{ALC}$-concepts $C$ and $D$, we have $C \sqsubseteq_{\mathcal{K}} D$ iff $C \sqsubseteq_{\mathcal{T}} D$.
*Hint:* Use disjoint unions.