



## Description Logics

Winter Semester 2016

### Exercise Sheet 4

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**Exercise 4.15** In the lecture, we defined bisimulations for  $\mathcal{ALC}$ -concepts s.t. they capture the expressive power of  $\mathcal{ALC}$ , i.e. that bisimulation invariance for  $\mathcal{ALC}$ -concepts follows.

- (a) Extend the notion of a bisimulation relation to  $\mathcal{ALCN}$  s.t. bisimulation invariance for  $\mathcal{ALCN}$ -concepts follows.
- (b) Show bisimulation invariance for the bisimulation relation defined in exercise (a).
- (c) Prove that  $\mathcal{ALCQ}$  is more expressive than  $\mathcal{ALCN}$ .

**Exercise 4.16** Prove or refute the following claim:

If an  $\mathcal{ALC}$ -concept  $C$  is satisfiable w.r.t. an  $\mathcal{ALC}$ -TBox  $\mathcal{T}$ , then for all  $n \geq 1$  there is a model  $\mathcal{I}_n$  of  $\mathcal{T}$  such that:  $|C^{\mathcal{I}_n}| \geq n$ .

**Exercise 4.17** Prove that bisimulations are closed under

- (a) composition  $\circ$ , and
- (b) union  $\cup$ .

**Exercise 4.18** Prove or refute the following claim:

Given an  $\mathcal{ALC}$ -concept  $C$  and an  $\mathcal{ALC}$ -TBox  $\mathcal{T}$ . If  $\mathcal{I}$  is an interpretation and  $\mathcal{J}$  its filtration w.r.t.  $\text{sub}(C) \cup \text{sub}(\mathcal{T})$ , then the relation  $\rho = \{(d, [d]) \mid d \in \Delta^{\mathcal{I}}\}$  is a bisimulation.