TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Description Logics

Winter Semester 2016

Exercise Sheet 5

21st November 2016

PD Dr.-Ing. habil. Anni-Yasmin Turhan & İsmail İlkan Ceylan

Exercise 5.19 Use the tableau algorithm from the lecture to decide whether the following subsumption holds:

$$\neg\forall r.A \sqcap \forall r.C \sqsubseteq_{\mathcal{T}} \forall r.E$$

where $\mathcal{T} = \{C \equiv (\exists r.\neg B) \sqcap \neg A, \quad D \equiv \exists r.B, \quad E \equiv \neg(\exists r.A) \sqcap \exists r.D\}$.

Exercise 5.20 Extend the proof of Lemma 4.1 (local correctness) to the $\sqcap$-rule and the $\forall$-rule.

Exercise 5.21 Consider the tableau algorithm from the lecture and extend it with the following two rules:

- *Condition:* $\mathcal{A}$ contains $(\geq n\ r)(a)$, but $k = |\{b \mid r(a,b) \in \mathcal{A}\}| < n$
  Action: $\mathcal{A}' := \mathcal{A} \cup \{r(a,b_i) \mid k < i \leq n\}$ where b_i are new individual names
- *Condition:* $\mathcal{A}$ contains $(\leq n\ r)(a)$ and $k = |\{b \mid r(a,b) \in \mathcal{A}\}| > n$
  Action: $\mathcal{A}' := \mathcal{A} \cup \{A(b), \neg A(b)\}$ where A is a concept name and b is a new individual name

Is the obtained algorithm sound and complete for $\mathcal{ALCN}$? Explain why.

Exercise 5.22 Prove Lemma 4.5 from the lecture.

Let $\mathcal{A} \in \mathcal{M}$ where $\mathcal{A}_0 \xrightarrow{*} \mathcal{M}$.

(a) If $C(a) \in \mathcal{A}$, then $C \in \mathrm{Sub}(\mathcal{A}_0)$.

(b) If $r(a,x) \in \mathcal{A}$ and x is a new individual, then $\max_{C(a)\in\mathcal{A}} |C| > \max_{C(x)\in\mathcal{A}} |C|$.

Hint: Use induction on the number of rule applications.