TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Description Logics

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Exercise Sheet 6

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Exercise 6.23 Prove Lemma 4.6 from the lecture.

(a) $|\text{Sub}(\mathcal{A}_0)| \leq \widehat{m}$

(b) $|C| \leq \widehat{m}$ for all $C \in \text{Sub}(\mathcal{A}_0)$

(c) If $\mathcal{A}$ is an ABox obtained by applying the tableau rules to $\mathcal{A}_0$ and x is an individual occuring in $\mathcal{A}$, then $\mathtt{depth}_{\mathcal{A}}(x) \leq \widehat{m}$.

Exercise 6.24 The tableau algorithm for checking consistency of $\mathcal{ALC}$-ABoxes w.r.t. general TBoxes can be extended to inverse roles by adapting the $\exists$-rule and $\forall$-rule as follows: Let C be an $\mathcal{ALCI}$-concept, and r an $\mathcal{ALCI}$-role, i.e. r denotes a role or an inverse role name, and $(r^{-1})^{-1} = r$ holds.

$\exists$-rule: Condition: $\mathcal{A}$ contains $(\exists r.C)(a)$, a is not blocked, but there is no b with either $\{r(a,b), C(b)\} \subseteq \mathcal{A}$ or $\{r^{-1}(b,a), C(b)\} \subseteq \mathcal{A}$

Action: $\mathcal{A}' := \mathcal{A} \cup \{r(a,b), C(b)\}$ for a new individual b not occuring in $\mathcal{A}$

$\forall$-rule: Condition: $(\forall r.C)(a) \in \mathcal{A}$ and $r(a,b) \in \mathcal{A}$ or $r^{-1}(b,a) \in \mathcal{A}$, but $C(b) \notin \mathcal{A}$

Action: $\mathcal{A}' := \mathcal{A} \cup \{C(b)\}$

(a) Which blocking condition needs to be introduced to obtain a correct decision procedure?

(b) Is the extended tableau algorithm for $\mathcal{ALCI}$ sound and complete?

Exercise 6.25 Let $\mathcal{K} = \langle \mathcal{T}, \mathcal{A}_0 \rangle$ be an $\mathcal{ALC}$-knowledge base, where $\mathcal{T}$ is a general TBox. The *precompletion of* $\mathcal{K}$ is the set of ABoxes $\mathcal{M}$ that is produced by the tableau algorithm when starting with the set of ABoxes $\{\mathcal{A}_0\}$ and exhaustively applying all tableau rules except the modified $\exists$-rule.

(a) Show that $\mathcal{K}$ is consistent iff there is an open ABox $\mathcal{A} \in \mathcal{M}$ such that for all individual names a occurring in $\mathcal{A}$, the concept $C^a_{\mathcal{A}} := \bigsqcap_{C(a)\in\mathcal{A}} C$ is satisfiable w.r.t. $\mathcal{T}$.

Hint: For the "if" direction, proceed as follows: The correctness of the tableau algorithm for $\mathcal{ALC}$ implies that, if $C^a_{\mathcal{A}}$ is satisfiable, then exhaustively applying all (!) rules to the set of ABoxes $\{\{C^a_{\mathcal{A}}(a)\}\}$ yields a set $\mathcal{M}'$ that contains an open and complete ABox. Show how to join all these ABoxes to obtain an open and complete tableau for $\mathcal{A}$ and conclude that $\mathcal{A}_0$ is consistent w.r.t. $\mathcal{T}$.

(b) Use the result from a) to prove that ABox consistency in $\mathcal{ALC}$ can be decided in deterministic exponential time.