Description Logics

Winter Semester 2016

Exercise Sheet 7

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Exercise 7.26 For each of the following languages of binary $\{a, b\}$-trees, define a looping tree automaton that accepts it.

(a) The set of all trees that contain a branch (starting at the root) in which all nodes are labelled with a.

(b) The set of all trees that do not contain nodes n_0, n_1, n_2 such that

- $n_1 = n_0 i$ for some $i \in \{0, 1\}$,
- $n_2 = n_1 j$ for some $j \in \{0, 1\}$, and
- $T(n_0) = T(n_1) = T(n_2) = a$.

Exercise 7.27 Recall the following: A *propositional Horn clause* is of the form $p_1, \ldots, p_k \to p$ where $p_1, \ldots, p_k$ are propositional variables and p is a propositional variable or $\bot$. A *propositional Horn formula* is a finite set of propositional Horn clauses. The satisfiability problem of propositional Horn formulas can be decided in linear time.

Show that the emptiness problem for looping tree automata can be decided in linear time by giving a linear-time reduction to the satisfiability problem of propositional Horn formulas.

Exercise 7.28 For each of the following $\mathcal{ALC}$-concept descriptions C and $\mathcal{ALC}$-TBoxes $\mathcal{T}$ decide whether C is satisfiable w.r.t. $\mathcal{T}$ by constructing the looping tree automaton $\mathcal{A}_{C,\mathcal{T}}$ and checking its accepted language $L(\mathcal{A}_{C,\mathcal{T}})$ for emptiness.

(a) $C := A$
$\mathcal{T} := \{A \sqsubseteq \neg A\}$

(b) $C := A \sqcap \exists r.A$
$\mathcal{T} := \{A \sqsubseteq \forall r.\neg A\}$