Description Logics

Winter Semester 2016

Exercise Sheet 11

16th January 2017

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Exercise 11.37 The description logic $\mathcal{S}$ extends $\mathcal{ALC}$ with *transitivity axioms* $\mathsf{trans}(r)$ for roles $r \in N_R$. The semantics are defined as follows: $\mathcal{I} \models \mathsf{trans}(r)$ if, and only if, $r^{\mathcal{I}}$ is transitive. Furthermore, an $\mathcal{S}$-knowledge base $(\mathcal{T}, \mathcal{A}, \mathcal{R})$ consists of an $\mathcal{ALC}$-knowledge base $(\mathcal{T}, \mathcal{A})$ and a *RBox* $\mathcal{R}$ of transitivity axioms. Prove the following claims:

(a) $\mathsf{trans}(r)$ cannot be expressed in $\mathcal{ALC}$, i.e., $\mathcal{S}$ is more expressive than $\mathcal{ALC}$.
Hint: Show that the FOL-formula $\forall x.\forall y.\forall z.(R(x,y) \wedge R(y,z)) \rightarrow R(x,z)$ is not equivalent to a formula in the two-variable-fragment of FOL.

(b) For an arbitrary TBox $\mathcal{T}$, the concept description $C_{\mathcal{T}}$ is defined as $\bigsqcap_{C \sqsubseteq D \in \mathcal{T}} \neg C \sqcup D$. Then $\mathcal{T}$ and $\{\top \sqsubseteq C_{\mathcal{T}}\}$ have the same models.

(c) Let $\mathcal{K} = (\mathcal{T}, \mathcal{A}, \mathcal{R})$ be a $\mathcal{S}$-knowledge base such that w.l.o.g. $\mathcal{T}$ consists of a single GCI $\top \sqsubseteq C_{\mathcal{T}}$, and $C_{\mathcal{T}}$ is in NNF. Define the $\mathcal{ALC}$-knowledge base $\mathcal{K}^+ := (\mathcal{T}^+, \mathcal{A})$ where $\mathcal{T}^+ := \mathcal{T} \cup \{\forall r.C \sqsubseteq \forall r.\forall r.C \mid \mathsf{trans}(r) \in \mathcal{R}, \forall r.C \in \mathrm{Sub}(C_{\mathcal{T}})\}$.
Then $\mathcal{K}$ is consistent if, and only if, $\mathcal{K}^+$ is consistent.
Consequently, the tableaux algorithm for $\mathcal{ALC}$ can also be utilized for $\mathcal{S}$.

(d) The problem of deciding consistency of a $\mathcal{S}$-knowledge base (with a general TBox) is ExpTime-complete.

Exercise 11.38 Let $f_1, \dots, f_m$ and $g_1, \dots, g_n$ be (not necessarily distinct) abstract features. A *feature agreement* is a concept of the form $(f_1 \circ \cdots \circ f_m) \downarrow (g_1 \circ \cdots \circ g_n)$ with the semantics:

$$\big((f_1 \circ \cdots \circ f_m) \downarrow (g_1 \circ \cdots \circ g_n)\big)^{\mathcal{I}} := \big\{d \in \Delta^{\mathcal{I}} \mid f_m^{\mathcal{I}}(\cdots f_2^{\mathcal{I}}(f_1^{\mathcal{I}}(d)) \cdots) = g_n^{\mathcal{I}}(\cdots g_2^{\mathcal{I}}(g_1^{\mathcal{I}}(d)) \cdots)\big\}$$

Feature disagreements ($\uparrow$) are defined analogously. The description logic $\mathcal{ALCF}$ extends $\mathcal{ALC}$ with feature agreements and feature disagreements. Show that satisfiability w.r.t. general TBoxes is undecidable for $\mathcal{ALCF}$.

Hint. Use a reduction from the domino problem presented in the lecture.