Description Logic

Winter Semester 2017/18

Exercise Sheet 1

18th October 2017

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Exercise 1.1 When solving reasoning problems in knowledge representation, we are often interested in *decision problems*. These are problems that can be answered with "yes" and "no" such as "Does TBox $\mathcal{T}$ have a model?". When solving decision problems, we are usually interested in algorithms that are decision procedures, i.e., which are

- *sound*: whenever the algorithm stops answering "yes", this answer is correct;
- *complete*: whenever the algorithm stops answering "no", this answer is correct; and
- *terminating*: the algorithm stops after finite time on every possible input.

Assume that we use *propositional logic* for knowledge representation and that the reasoning problem we have to solve is the following: given a formula ϕ, decide whether ϕ is satisfiable. Which of the following algorithms is sound/complete/terminating? What is the time consumption of the algorithms?

(a) Always return "yes".

(b) Always return "no".

(c) Enter an infinite loop, never return.

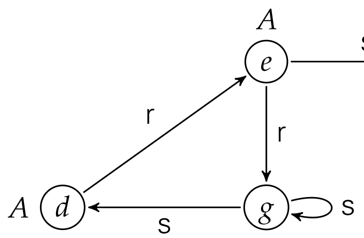
(d) Using a Hilbert-style calculus, enumerate all valid formulas of propositional logic. If $\neg\phi$ is among them, return "no". Otherwise, continue.

(e) Enumerate all truth assignments for the variables in ϕ. For each truth assignment, check whether it satisfies ϕ. If a satisfying truth assignment is found, return "yes". Otherwise return "no".

(f) Convert ϕ into disjunctive normal form. If every (conjunctive) clause contains two literals of the form x and $\neg x$, return "no". Otherwise, return "yes".

(g) Non-deterministically guess a truth assignment for ϕ. Check whether it satisfies ϕ, accept if it does and reject otherwise.

Exercise 1.2 Consider the (graphical representation of the) interpretation $\mathcal{I}$ with $\Delta^{\mathcal{I}} = \{d, e, f, g\}$:



For each of the following $\mathcal{ALCNI}$-concepts C, list all elements x of $\Delta^{\mathcal{I}}$ such that $x \in C^{\mathcal{I}}$:

(a) $A \sqcup B$

(b) $\exists s.\neg A$

(c) $\forall s.A$

(d) $(\geq 2\, s)$

(e) $\exists s.\exists s.\exists s.\exists s.A$

(f) $\forall s^{-}.\exists s.\exists s.\exists s.A$

(g) $\neg\exists r.(\neg A \sqcap \neg B)$

(h) $\exists s.(A \sqcap \forall s.\neg B) \sqcap \neg\forall r.\exists r.(A \sqcup \neg A)$

Exercise 1.3 Build an $\mathcal{ALC}$-TBox $\mathcal{T}$ that captures each of the following statements in a suitable GCI or concept definition, using only the concept names

$$\text{Vehicle, Boat, Bicycle, Car, Device, Wheel, Engine, Axle,}$$
$$\text{Rotation, Water, Human, Driver, Adult, Child}$$

and the role names

$$\text{hasPart, poweredBy, capableOf, travelsOn, controls.}$$

(a) Cars are exactly those vehicles that have wheels and are powered by an engine.

(b) Bicycles are exactly those vehicles that have wheels and are powered by a human.

(c) Boats are exactly those vehicles that travel on water.

(d) Boats have no wheels.

(e) Cars and bicycles do not travel on water.

(f) Wheels are exactly those devices that have an axle and are capable of rotation.

(g) Drivers are exactly those humans who control a vehicle.

(h) Drivers of cars are adults.

(i) Humans are not vehicles.

(j) Wheels or engines are not humans.

(k) Humans are either adults or children.

(l) Adults are not children.

Exercise 1.4 Draw a model $\mathcal{I}$ of the TBox $\mathcal{T}$ from Exercise 1.3 in which $\text{Driver}^{\mathcal{I}}$ is not empty. Modify it such that it is no longer a model, in three different ways.

Exercise 1.5 Extend the mapping π_x of $\mathcal{ALC}$-concept descriptions to first-order formulas given in the lecture to the description logic $\mathcal{ALCQ}$, which augments $\mathcal{ALC}$ with qualified number restrictions.

Exercise 1.6 Recall that the description logic $\mathcal{ALC}$ is equipped with the concept constructors negation ($\neg$), conjunction ($\sqcap$), disjunction ($\sqcup$), existential restriction ($\exists r.C$), and universal restriction ($\forall r.C$). Each subset of this set of constructors gives rise to a fragment of $\mathcal{ALC}$.

Identify all minimal fragments that are equivalent to $\mathcal{ALC}$ in the sense that, for every $\mathcal{ALC}$-concept, there is an equivalent concept in the fragment. (Two concepts are equivalent iff they have the same extension in every interpretation.)