TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Description Logic

Winter Semester 2017/18

Exercise Sheet 2

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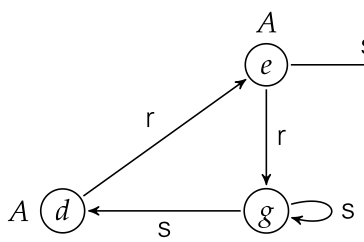
Exercise 2.1 Consider the ABox

$$\mathcal{A} = \{A(d),\ A(e),\ A(f),\ B(f),\ r(d,e),\ r(e,g),\ s(e,f),\ s(g,g),\ s(g,d)\}$$

with the following graphical representation:



For each of the following $\mathcal{ALC}$-concepts C, list all individuals that are instances of C w.r.t. $\mathcal{A}$. Compare your results to Exercise 1.2.

(a) $A \sqcup B$

(b) $\exists s.\neg A$

(c) $\forall s.A$

(d) $\exists s.\exists s.\exists s.\exists s.A$

(e) $\neg\exists r.(\neg A \sqcap \neg B)$

(f) $\exists s.(A \sqcap \forall s.\neg B) \sqcap \neg\forall r.\exists r.(A \sqcup \neg A)$

Exercise 2.2 Extend the $\mathcal{ALC}$-TBox $\mathcal{T}$ from Exercise 1.3 to an $\mathcal{ALC}$-knowledge base $\mathcal{K}$ by adding axioms that capture the following statements (using the additional concept name Broken and the individual names Bob and $\mathrm{QE2}$):

(i) Cars have between three and four wheels.

(ii) Bicycles have exactly two wheels.

(iii) A vehicle is controlled by at most one human.

(iv) A thing with a broken part is broken.

(v) Bob controls a car with a wheel that has a broken axle.

(vi) Bob is a human.

(vii) Bob controls QE2.

(viii) QE2 is a vehicle that travels on water.

Which of the following statements about $\mathcal{K}$ is true?

(a) $\mathcal{K}$ is consistent.

(b) The concept $\mathsf{Boat} \sqcap \exists \mathsf{hasPart}.\mathsf{Wheel}$ is satisfiable w.r.t. $\mathcal{K}$.

(c) The concept $\mathsf{Boat} \sqcap \exists \mathsf{poweredBy}.\mathsf{Engine}$ is satisfiable w.r.t. $\mathcal{K}$.

(d) The concept $\mathsf{Car} \sqcap \mathsf{Bicycle}$ is satisfiable w.r.t. $\mathcal{K}$.

(e) The concept $\mathsf{Driver} \sqcap \mathsf{Vehicle}$ is satisfiable w.r.t. $\mathcal{K}$.

(f) The concept $\mathsf{Driver} \sqcap \mathsf{Child}$ is satisfiable w.r.t. $\mathcal{K}$.

(g) The concept $\exists \mathsf{controls}.\mathsf{Car} \sqcap \mathsf{Child}$ is satisfiable w.r.t. $\mathcal{K}$.

(h) The concept $\exists \mathsf{controls}.\mathsf{Car} \sqcap \mathsf{Child} \sqcap \mathsf{Human}$ is satisfiable w.r.t. $\mathcal{K}$.

(i) Bob is an instance of Adult w.r.t. $\mathcal{K}$.

(j) Bob is an instance of Driver w.r.t. $\mathcal{K}$.

(k) Bob is an instance of $(\mathsf{Adult} \sqcap \mathsf{Driver})$ w.r.t. $\mathcal{K}$.

(l) Bob is an instance of $\exists \mathsf{controls}.(\mathsf{Car} \sqcap \mathsf{Broken})$ w.r.t. $\mathcal{K}$.

(m) $\mathsf{QE2}$ is an instance of Boat w.r.t. $\mathcal{K}$.

(n) Driver is subsumed by Human w.r.t. $\mathcal{K}$.

(o) Adult is subsumed by Human w.r.t. $\mathcal{K}$.

(p) $\mathsf{Human} \sqcap \exists \mathsf{controls}.(\mathsf{Vehicle} \sqcap \exists \mathsf{hasPart}.\mathsf{Wheel} \sqcap \exists \mathsf{poweredBy}.\mathsf{Engine})$ is subsumed by Adult w.r.t. $\mathcal{K}$.

(q) $\exists \mathsf{controls}.\mathsf{Car}$ is subsumed by Adult w.r.t. $\mathcal{K}$.

Exercise 2.3 Prove Lemma 2.18 from the lecture: Let $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ be a knowledge base. If a is an instance of C w.r.t. $\mathcal{K}$ and $C \sqsubseteq_{\mathcal{T}} D$, then a is an instance of D w.r.t. $\mathcal{K}$.

Exercise 2.4 Consider the acyclic TBox $\mathcal{T}$ that contains the following definitions:

$$\begin{aligned}
\mathsf{MastersStudent} &\equiv \mathsf{Student} \sqcap \exists \mathsf{enrolledIn}.\mathsf{MastersProgram} \\
\mathsf{DiplomStudent} &\equiv \mathsf{Student} \sqcap \exists \mathsf{enrolledIn}.\mathsf{DiplomProgram} \\
\mathsf{Student} &\equiv \mathsf{Human} \sqcap \exists \mathsf{attends}.\mathsf{Course} \\
\mathsf{Lecturer} &\equiv \mathsf{Human} \sqcap \exists \mathsf{teaches}.\mathsf{Course} \\
\mathsf{Course} &\equiv \mathsf{Lecture} \sqcup \mathsf{Seminar} \\
\mathsf{Human} &\equiv \mathsf{Woman} \sqcup \mathsf{Man}
\end{aligned}$$

Construct its expanded version $\widehat{\mathcal{T}}$, as defined in the lecture (proof of Proposition 2.7). After each replacement step, write down the dependency graph $G_{\mathcal{T}'}$ and the function $\ell_{\mathcal{T}'}$ for the current TBox $\mathcal{T}'$.

Exercise 2.5 *Cyclic TBoxes* are defined similarly to acyclic TBoxes, with the exception that they contain cyclic definitions. We consider an expansion procedure for cyclic TBoxes that applies the same replacement steps as for acyclic TBoxes, but does not require that $\ell_{\mathcal{T}}(A) = 1$ to apply a replacement step to A. This procedure obviously does not terminate.

(a) Show that applying one such replacement step to a cyclic TBox $\mathcal{T}$ may even be incorrect, i.e., the resulting TBox may not be equivalent to $\mathcal{T}$.

(b) Find a cyclic TBox for which all replacement steps are correct (even though the procedure does not terminate).

Exercise 2.6 Recall Corollary 2.8 from the lecture, which shows that every primitive interpretation has a unique extension to a model of a given acyclic TBox. Consider the *cyclic* TBox

$$\mathcal{T} := \{A \equiv \exists r.B,\ B \equiv \exists r.\neg A\}.$$

(a) Is there a primitive interpretation that cannot be extended to a model of $\mathcal{T}$?

(b) Is there a primitive interpretation that can be extended in several ways to a model of $\mathcal{T}$?