TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Description Logic

Winter Semester 2017/18

Exercise Sheet 3

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Exercise 3.1 In the lecture, we defined bisimulations for $\mathcal{ALC}$-concepts and showed bisimulation invariance of $\mathcal{ALC}$.

(a) Define a notion of "$\mathcal{ALCN}$-bisimulation" that is appropriate for $\mathcal{ALCN}$ in the sense that bisimilar elements satisfy the same $\mathcal{ALCN}$-concepts.

(b) Use this definition to show that $\mathcal{ALCQ}$ is more expressive than $\mathcal{ALCN}$.

Exercise 3.2 Since bisimulations are binary relations, one can apply standard operations, such as composition ($\circ$), union ($\cup$), and intersection ($\cap$), to them. Prove that the class of bisimulations is closed under composition and union, but not under intersection.

Exercise 3.3 Recall Theorem 3.8 from the lecture, which says that the disjoint union of a family of models of an $\mathcal{ALC}$-TBox $\mathcal{T}$ is a again a model of $\mathcal{T}$. Note that the disjoint union is only defined for concept and role names.

Extend the notion of disjoint union to individual names such that the following holds: For any family $(\mathcal{I}_\nu)_{\nu\in\mathfrak{N}}$ of models of an $\mathcal{ALC}$-knowledge base $\mathcal{K}$, the disjoint union $\biguplus_{\nu\in\mathfrak{N}}\mathcal{I}_\nu$ is also a model of $\mathcal{K}$.

Exercise 3.4 Let $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ be a consistent $\mathcal{ALC}$-knowledge base. We write $C \sqsubseteq_\mathcal{K} D$ if $C^\mathcal{I} \subseteq D^\mathcal{I}$ holds for every model $\mathcal{I}$ of $\mathcal{K}$. Prove that for all $\mathcal{ALC}$-concepts C and D we have $C \sqsubseteq_\mathcal{K} D$ iff $C \sqsubseteq_\mathcal{T} D$.

Hint: Use the modified definition of disjoint union from the previous exercise.