TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Description Logic

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Exercise Sheet 7

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Exercise 7.1 We consider another form of blocking, where an individual can be blocked by an individual that is not necessarily an ancestor: *anywhere blocking*. Instead of the ancestor relation, it uses the age of an individual to determine the blocking relation.

The *age* of an individual a, denoted by $\mathsf{age}(a)$, is defined as 0 for individuals that occur in the input ABox $\mathcal{A}$ and as n for a new individual that was generated by the nth application of the $\exists$-rule.

Let $\mathcal{A}'$ be an ABox obtained by applying the tableau rules of $\mathsf{consistent}(\mathcal{T}, \mathcal{A})$ for general TBoxes. A tree individual b is *anywhere blocked by an individual* a in $\mathcal{A}'$ if

- $\mathsf{con}_{\mathcal{A}'}(b) \subseteq \mathsf{con}_{\mathcal{A}'}(a)$,
- $\mathsf{age}(a) < \mathsf{age}(b)$, and
- a is not blocked.

As before, the descendants of b are then also considered blocked.

Prove that the tableau algorithm with anywhere blocking is a decision procedure for consistency of $\mathcal{ALC}$-knowledge bases with general TBoxes.

Exercise 7.2 Let $\mathcal{K} = (\mathcal{T}, \mathcal{A}_0)$ be an $\mathcal{ALC}$-knowledge base, where $\mathcal{T}$ is a general TBox. A *precompletion* of $\mathcal{K}$ is a clash-free ABox $\mathcal{A}$ that is obtained from $\mathcal{K}$ by exhaustively applying all expansion rules except the $\exists$-rule.

Show that $\mathcal{K}$ is consistent iff there is a precompletion $\mathcal{A}$ of $\mathcal{K}$ such that, for all individual names a occurring in $\mathcal{A}$, the concept $C^a_{\mathcal{A}} := \bigsqcap_{a:C\in\mathcal{A}} C$ is satisfiable w.r.t. $\mathcal{T}$.

Exercise 7.3 Let C be an $\mathcal{ALC}$-concept. We denote by $\#C$ the number of occurrences of the constructors $\sqcup$, $\sqcap$, $\exists$, and $\forall$ within C. The multiset $M(C)$ contains, for each occurrence of a subconcept of the form $\neg D$ in C, the number $\#D$.

Use this representation to prove that exhaustively applying the following transformation rules to an $\mathcal{ALC}$-concept always terminates, regardless of the order of rule applications:

$$
\begin{aligned}
\neg(C \sqcap D) &\rightsquigarrow \neg\neg\neg C \sqcup \neg\neg\neg D \\
\neg(C \sqcup D) &\rightsquigarrow \neg\neg\neg C \sqcap \neg\neg\neg D \\
\neg\neg C &\rightsquigarrow C \\
\neg(\exists r.C) &\rightsquigarrow \forall r.\neg C \\
\neg(\forall r.C) &\rightsquigarrow \exists r.\neg C
\end{aligned}
$$