Description Logic

Winter Semester 2017/18

Exercise Sheet 8

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Exercise 8.1 Recall the tableau algorithm for $\mathcal{ALCN}$ with blocking, and the example that shows non-termination when using the $\leq$-rule *without pruning*. At each step of the algorithm, write down the multiset $\mu(\mathcal{A})$ for the current ABox $\mathcal{A}$, and explain why the termination proof fails in this case.

Exercise 8.2 Prove that the tableau algorithm for $\mathcal{ALCN}$ is sound and complete.

Exercise 8.3 We extend the tableau algorithm from $\mathcal{ALCN}$ to $\mathcal{ALCQ}$ by modifying the $\geq$-rule and the $\leq$-rule as follows:

The $\geq$-rule	
Condition:	$\mathcal{A}$ contains $a:(\geq n\,r.C)$, but there are no n distinct individuals $b_1,\dots,b_n$ with $\{(a,b_i):r,\ b_i:C \mid 1 \leq i \leq n\} \subseteq \mathcal{A}$, and a is not blocked
Action:	$\mathcal{A} \longrightarrow \mathcal{A} \cup \{(a,d_i):r,\ d_i:C \mid 1 \leq i \leq n\} \cup \{d_i \neq d_j \mid 1 \leq i < j \leq n\}$, where $d_1,\dots,d_n$ are new individual names

The $\leq$-rule	
Condition:	$\mathcal{A}$ contains $a:(\leq n\,r.C)$, and there are $n+1$ distinct individuals $b_0,\dots,b_n$ with $\{(a,b_i):r,\ b_i:C \mid 0 \leq i \leq n\} \subseteq \mathcal{A}$
Action:	$\mathcal{A} \longrightarrow \mathsf{prune}(\mathcal{A},b_j)[b_j \mapsto b_i] \cup \{b_i = b_j\}$ for $i \neq j$ such that, if b_j is a root individual, then so is b_i

For the knowledge base

$$\big(\{C \sqsubseteq E\},\ \{a:(\leq 1\,r.(D \sqcap E)),\ (a,b):r,\ b:C \sqcap D,\ (a,c):r,\ c:D \sqcap E,\ c:\neg C\}\big),$$

determine whether it is consistent, and whether the proposed algorithm detects this.

Exercise 8.4 Let $\mathcal{T}$ be an acyclic TBox in NNF, and let $\mathcal{T}^{\sqsubseteq}$ be obtained from $\mathcal{T}$ by replacing each definition $A \equiv C \in \mathcal{T}$ with $A \sqsubseteq C$. Prove that every concept name A_0 is satisfiable w.r.t. $\mathcal{T}$ iff it is satisfiable w.r.t. $\mathcal{T}^{\sqsubseteq}$. Does this also hold for the acyclic TBox $\{A \equiv C \sqcap \neg B,\ B \equiv P,\ C \equiv P\}$?

Exercise 8.5 Use the $\mathcal{ALC}$-Worlds algorithm to decide satisfiability of the concept name A_0 w.r.t. the following simple TBox:

$$\begin{aligned}
\{\ & A_0 \equiv A_1 \sqcap A_2, && A_1 \equiv \exists r.A_3, && A_3 \equiv P,\\
& A_2 \equiv A_4 \sqcap A_5, && A_4 \equiv \exists r.A_6, && A_6 \equiv Q,\\
& A_5 \equiv A_7 \sqcap A_8, && A_7 \equiv \forall r.A_4, && A_8 \equiv \forall r.A_9,\\
& A_9 \equiv \forall r.A_{10}, && A_{10} \equiv \neg P\ \}
\end{aligned}$$

Draw the recursion tree of a successful run and of an unsuccessful run. Does the algorithm return a positive or a negative result on this input?