



Description Logic

Winter Semester 2017/18

Exercise Sheet 10

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Exercise 10.1 Determine whether Player 2 has a winning strategy in the following infinite Boolean games, where the initial configuration t_0 assigns *false* to all variables:

- (a) $\varphi = (p_1 \wedge p_2 \wedge \neg q_1) \vee (p_3 \wedge p_4 \wedge \neg q_2) \vee (\neg(p_1 \vee p_4) \wedge q_1 \wedge q_2)$ with $\Gamma_1 = \{p_1, p_2, p_3, p_4\}$ and $\Gamma_2 = \{q_1, q_2\}$
- (b) $\varphi = ((p_1 \leftrightarrow \neg q_1) \wedge (p_2 \leftrightarrow \neg q_2) \wedge (p_1 \leftrightarrow p_2)) \vee ((p_1 \leftrightarrow q_1) \wedge (p_2 \leftrightarrow q_2) \wedge (p_1 \leftrightarrow \neg p_2))$ with $\Gamma_1 = \{p_1, p_2\}$ and $\Gamma_2 = \{q_1, q_2\}$

Exercise 10.2 Are the following variations of infinite Boolean games also EXPTIME-hard?

- (a) Player 1 wins if the constructed truth assignment falsifies the formula φ , instead of satisfying it.
- (b) Player 2 starts instead of Player 1.
- (c) The variables are not assigned to a specific player; instead, the active player can choose any variable and assign it a new truth value; variables can be chosen multiple times.
- (d) The two players must always flip the assignment of a variable, i.e., the truth assignment cannot be left unchanged.

Exercise 10.3 The *universal role* is a role name u whose interpretation is fixed as $\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ in any interpretation \mathcal{I} . Let \mathcal{ALC}^u denote the extension of \mathcal{ALC} with the universal role. Show that concept satisfiability in \mathcal{ALC}^u without TBoxes is EXPTIME-complete.