Exercise 11.1  Let $\mathcal{T}_1$ be an $\mathcal{EL}$-TBox, $C, D \in \mathcal{EL}$-concepts, and $\mathcal{T}_2 = \mathcal{T}_1 \cup \{A \sqsubseteq C, D \sqsubseteq B\}$, where $A, B$ are new concept names (as in Lemma 6.1). Show that $\mathcal{T}_2$ is a conservative extension of $\mathcal{T}_1$. Is this still the case after adding $A \sqsubseteq B$ to $\mathcal{T}_2$? What about adding $B \sqsubseteq A$?

Exercise 11.2  Consider the TBox

$$\mathcal{T} = \{A \sqsubseteq B \cap \exists r.C, B \sqcap \exists r.B \sqsubseteq C \sqcap D, C \sqsubseteq (\exists r.A) \cap B, (\exists r.\exists r.B) \cap D \sqsubseteq \exists r.(A \sqcap B)\},$$

where $A, B, C, D$ are concept names. Use the classification procedure for $\mathcal{EL}$ to check whether the following subsumption relationships hold w.r.t. $\mathcal{T}$:

(a) $A \sqsubseteq B$
(b) $A \sqsubseteq \exists r.\exists r.A$
(c) $B \sqcap \exists r.A \sqsubseteq \exists r.C$