Description Logic

Winter Semester 2017/18

Exercise Sheet 12

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Exercise 12.1 We consider simulations, which are "one-sided" variants of bisimulations. Given interpretations $\mathcal{I}_1$ and $\mathcal{I}_2$, the relation $\sigma \subseteq \Delta^{\mathcal{I}_1} \times \Delta^{\mathcal{I}_2}$ is a *simulation* between $\mathcal{I}_1$ and $\mathcal{I}_2$ if

- whenever $d_1 \; \sigma \; d_2$ and $d_1 \in A^{\mathcal{I}_1}$, then $d_2 \in A^{\mathcal{I}_2}$, for all $d_1 \in \Delta^{\mathcal{I}_1}$, $d_2 \in \Delta^{\mathcal{I}_2}$, and $A \in \mathbf{C}$;
- whenever $d_1 \; \sigma \; d_2$ and $(d_1, d_1') \in r^{\mathcal{I}_1}$, then there exists a $d_2' \in \Delta^{\mathcal{I}_2}$ such that $d_1' \; \sigma \; d_2'$ and $(d_2, d_2') \in r^{\mathcal{I}_2}$, for all $d_1, d_1' \in \Delta^{\mathcal{I}_1}$, $d_2 \in \Delta^{\mathcal{I}_2}$, and $r \in \mathbf{R}$.

We write $(\mathcal{I}_1, d_1) \precsim (\mathcal{I}_2, d_2)$ if there is a simulation σ between $\mathcal{I}_1$ and $\mathcal{I}_2$ such that $d_1 \; \sigma \; d_2$.

(a) Show that $(\mathcal{I}_1, d_1) \sim (\mathcal{I}_2, d_2)$ implies $(\mathcal{I}_1, d_1) \precsim (\mathcal{I}_2, d_2)$ and $(\mathcal{I}_2, d_2) \precsim (\mathcal{I}_1, d_1)$.

(b) Is the converse of the implication in (a) also true?

(c) Show that, if $(\mathcal{I}_1, d_1) \precsim (\mathcal{I}_2, d_2)$, then for all $\mathcal{EL}$-concepts C it holds that $d_1 \in C^{\mathcal{I}_1}$ implies $d_2 \in C^{\mathcal{I}_2}$.

(d) Which of the constructors disjunction, negation, or value restriction can be added to $\mathcal{EL}$ without losing the property in (c)?

(e) Show that $\mathcal{ALC}$ is more expressive than $\mathcal{EL}$.

(f) Show that $\mathcal{ELI}$ is more expressive than $\mathcal{EL}$.

(g) Can the fact that subsumption in $\mathcal{EL}$ is decidable in polynomial time, while subsumption in $\mathcal{ELI}$ is ExpTime-complete, be used to show that $\mathcal{ELI}$ is more expressive than $\mathcal{EL}$?

Exercise 12.2 Consider the TBox

$$\mathcal{T} = \{A_1 \sqcap A_2 \sqsubseteq \exists r.B,\ \exists r^-.A_2 \sqsubseteq C,\ A \sqsubseteq A_1 \sqcap A_2,\ \exists r.(B \sqcap C) \sqsubseteq D\},$$

where A, A_1, A_2, B, C, D are concept names. Use the classification procedure for $\mathcal{ELI}$ to check whether the following subsumption relationships hold w.r.t. $\mathcal{T}$:

(a) $A \sqsubseteq D$

(b) $\exists r.A \sqsubseteq \exists r.D$

(c) $A \sqsubseteq \exists r.A$