Exercise 12.1 We consider simulations, which are “one-sided” variants of bisimulations. Given interpretations $I_1$ and $I_2$, the relation $\sigma \subseteq \Delta^{I_1} \times \Delta^{I_2}$ is a simulation between $I_1$ and $I_2$ if

- whenever $d_1 \sigma d_2$ and $d_1 \in A^{I_1}$, then $d_2 \in A^{I_2}$, for all $d_1 \in \Delta^{I_1}$, $d_2 \in \Delta^{I_2}$, and $A \in C$;
- whenever $d_1 \sigma d_2$ and $(d_1,d'_1) \in r^{I_1}$, then there exists a $d'_2 \in \Delta^{I_2}$ such that $d'_1 \sigma d'_2$ and $(d_2,d'_2) \in r^{I_2}$, for all $d_1,d'_1 \in \Delta^{I_1}$, $d_2 \in \Delta^{I_2}$, and $r \in R$.

We write $(I_1,d_1) \sim (I_2,d_2)$ if there is a simulation $\sigma$ between $I_1$ and $I_2$ such that $d_1 \sigma d_2$.

(a) Show that $(I_1,d_1) \sim (I_2,d_2)$ implies $(I_1,d_1) \mathbin{\overset{\sim}{\Rightarrow}} (I_2,d_2)$ and $(I_2,d_2) \mathbin{\overset{\sim}{\Rightarrow}} (I_1,d_1)$.

(b) Is the converse of the implication in (a) also true?

(c) Show that, if $(I_1,d_1) \mathbin{\overset{\sim}{\Rightarrow}} (I_2,d_2)$, then for all $\mathcal{EL}$-concepts $C$ it holds that $d_1 \in C^{I_1}$ implies $d_2 \in C^{I_2}$.

(d) Which of the constructors disjunction, negation, or value restriction can be added to $\mathcal{EL}$ without losing the property in (c)?

(e) Show that $\mathcal{ALC}$ is more expressive than $\mathcal{EL}$.

(f) Show that $\mathcal{ELI}$ is more expressive than $\mathcal{EL}$.

(g) Can the fact that subsumption in $\mathcal{EL}$ is decidable in polynomial time, while subsumption in $\mathcal{ELI}$ is EXPTIME-complete, be used to show that $\mathcal{ELI}$ is more expressive than $\mathcal{EL}$?

Exercise 12.2 Consider the TBox

$$
\mathcal{T} = \{ A_1 \sqcap A_2 \sqsubseteq \exists r.B, \exists r^{-}.A_2 \sqsubseteq C, A \sqsubseteq A_1 \sqcap A_2, \exists r.(B \sqcap C) \sqsubseteq D \},
$$

where $A, A_1, A_2, B, C, D$ are concept names. Use the classification procedure for $\mathcal{ELI}$ to check whether the following subsumption relationships hold w.r.t. $\mathcal{T}$:

(a) $A \sqsubseteq D$

(b) $\exists r.A \sqsubseteq \exists r.D$

(c) $A \sqsubseteq \exists r.A$