Subsection 6.5
Inference relations based on preferential models
The idea of preferential models is to generalize the concept of minimal models that were used for circumscription.

Reasoning under preferential models semantics does no longer consider all models to compute consequences, but only preferred ones.

In the following we abstract from the underlying logic and use $\mathcal{L}$ and its elements (‘propositions’).
Definition 6.10 (preferential model structure)

A preferential model structure is a triple \((MS, \models, <)\), where

- \(MS\) is a set of models.
- \(\models \subseteq MS \times \mathcal{L}\) is a relation between models and formulas in \(\mathcal{L}\) and is called the satisfaction relation of the structure.
- \(< \subseteq MS \times MS\) is a relation on \(MS\) and is called the preference relation.

Let \(m \in MS\) and \(L \subseteq \mathcal{L}\).

Model \(m\) preferentially satisfies \(L\), denoted by \(m \models <\), iff \(m \models\) and there is no model \(m' \in MS\) s.t. \(m' < m\) and \(m' \models\). We call \(m\) a preferential model of \(L\).

The intuition for the

- satisfaction relation \(\models\) is that it states which formulas are satisfied by which models.
- preference relation \(<\) is that it states which models are preferred over which other model.
Inference based on preferential models

Definition 6.11 ($\models<, C<)$
Based on preferential models we define an inference relation $\models<$ determined by a preferential model structure $(MS, \models, <)$ as:

$$L \models< x \iff \text{for all } m \in MS, m \models< L \text{ implies } m \models x.$$ 

The inference operation $C< (L)$ determined by a preferential model structure $(MS, \models, <)$ is defined as follows:

$$C<(L) = \{x \in \mathcal{L} \mid \text{for all } m \in MS, m \models< L \text{ implies } m \models x\}.$$ 

Intuition:
Formula $x$ follows nonmonotonically from a set of formulas $L$ if it is satisfied by all preferential models of $L$. 

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Theorem 6.12

*Every preferential model structure satisfies Inclusion, Idempotence and Cut.*

Proof: blackboard
Pure conditions of preferential model structures

Preferential model structures do not satisfy Cumulativity.

**Example 6.13**

Let $MS$ be the infinite set $\{m_1, m_2, \ldots \}$, and define $m_i < m_j$ iff $j < i$. Let also $x, y, z$ be elements of the underlying language $L$ such that:

- $m_i \models x$ for all $i > 0$
- $m_i \not\models y$ for all $i > 0$
- $m_i \models z$ iff $i = 1$

There is no minimal model satisfying $x$, therefore $\{x\} \not\models y$ and $\{x\} \not\models z$. But $m_1$ is the minimal model satisfying $\{x, z\}$ and $m_1 \not\models y$. So, $\{x, z\} \not\models y$ is false and Cautious Monotony and thus Cumulativity is violated.