Subsection 4.3
Stable sets and their properties
Stable sets — origin

- Stable belief sets were introduced by Robert Stalnaker in the early ‘80s.
- Proposed as a formal representation of the epistemic state of an ideally rational agent, with full introspective capabilities.
- Assumes a propositional language, endowed with a modal operator $\square \varphi$ interpreted as “$\varphi$ is believed.”
- A set of formulas is a stable set if it is “stable” under classical inference and epistemic introspection.
- Influenced research on AE logics and nonmonotonic logics in general.
Stable sets — definition

Definition 4.7 (stable sets)
Let $E$ be a set of autoepistemic formulas. $E$ is called stable iff

- $E$ is deductively closed, i.e. $E = Th(E)$,
- $\varphi \in E$ implies $L\varphi \in E$, for all AE-formula $\varphi$, and
- $\varphi \notin E$ implies $\neg L\varphi \in E$, for all AE-formula $\varphi$

Note: Expansions are stable sets by definition. Thus they inherit all the properties we show for stable sets.
Stable sets and expansions

**Theorem 4.8**

For an AE-theory $T$ and a set of AE-formulas $E$ the following statements are equivalent:

1. $E$ is an expansion of $T$
2. $E$ is stable, $T \subseteq E$ and is $T$-sound.

Proof: blackboard
Lemma 4.9
For a stable set $E$ and an AE-formula $\varphi$ the following statements are equivalent:

a) $E \models_E \varphi$

b) $E \models \varphi$

c) $\varphi \in E$

For a FOL formula $\varphi$, the statements a)-c) are equivalent to

d) $E_0 \models \varphi$

Proof: blackboard
Stable sets are determined by their kernels

Stable sets are uniquely determined by their objective subsets, i.e. their kernels.

**Theorem 4.10**

For stable sets $E$ and $F$, $E_0 = F_0$ implies $E = F$.

Proof: blackboard
Existence of stable sets

How can expansions be computed? A first hint

Theorem 4.11

Let $T$ be a first order theory. Then there is a stable set $E$ with $E_0 = T$.

Proof: blackboard
Properties of stable sets

Theorem 4.12 (Orthogonality of stable sets)

Let $E$ and $F$ be different stable sets. Then $E \cup F$ is inconsistent.

Proof: blackboard

Theorem 4.13

Let $T$ be a first order theory. Then there is a stable set $E$ with $E_0 = T$.

Proof: blackboard