Independence in Logic and Algebra

George Metcalfe

Mathematical Institute, University of Bern, Switzerland
george.metcalfe@unibe.ch

Abstract

This talk will explore a notion of independence for formulas considered by De Jongh and Chagrova for intuitionistic propositional logic in [1] that is closely related to a notion of independence for elements of an algebraic structure studied by Marczewski and others in the 1950s [2]. Terms $t_1, \dots, t_n$ are said to be independent in a variety (equational class) V if any substitution mapping each variable x_i to t_i V-unifies only the equations in $x_1, \dots, x_n$ that are already satisfied by V. In [1], it is shown that this property is decidable for Heyting algebras, using Pitts' proof of uniform interpolation for intuitionistic propositional logic [4], and a description is given of independent pairs of formulas. Following [3], this talk will consider the problems of deciding and describing independence for several other case studies from logic and algebra, including groups, semigroups, lattices, modal algebras, and MV-algebras, and explain how independence relates to the notions of coherence and admissibility.

References

[1] Dick De Jongh and L. A. Chagrova. The decidability of dependency in intuitionistic propositional logic. *J. Symb. Log.*, 60(2):498–504, 1995.

[2] Edward Marczewski. A general scheme of the notions of independence in mathematics. *Bull. Acad. Polon. Sci.*, 6:731–736, 1958.

[3] George Metcalfe and Naomi Tokuda. Deciding dependence in logic and algebra. In *Dick de Jongh on Intuitionistic and Provability Logics*, volume 28 of *Outstanding Contributions to Logic*. Springer, 2024.

[4] Andrew M. Pitts. On an interpretation of second order quantification in first order intuitionistic propositional logic. *J. Symb. Log.*, 57(1):33–52, 1992.