TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Term Rewriting Systems

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Exercise Sheet 1 – Introductory Exercises

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Exercise 1.1 Consider the reduction system $(M, \rightarrow)$ with
$M := \{A_1, A_2, A_3, A_4, B_1, B_2, B_3, C_1, C_2, C_3, C_4, D, E\}$ and $\rightarrow \subseteq M \times M$ where

- $A_1 \rightarrow B_1$, $A_1 \rightarrow B_2$, $A_2 \rightarrow B_1$, $A_2 \rightarrow B_2$, $A_3 \rightarrow B_3$, $A_4 \rightarrow B_3$,
- $B_1 \rightarrow C_1$, $B_2 \rightarrow C_2$, $B_2 \rightarrow C_3$, $B_3 \rightarrow C_1$, $B_3 \rightarrow C_2$, $B_3 \rightarrow C_3$, $B_3 \rightarrow C_4$,
- $C_3 \rightarrow E$, $C_4 \rightarrow E$, and
- $D \rightarrow C_4$.

Answer the following questions.

(a) Which of the following properties are satisfied by $\rightarrow$? Justify your answer.

  (i) finite

  (ii) symmetric

  (iii) antisymmetric

  (iv) reflexive

  (v) irreflexive

  (vi) transitive

(b) Describe the closures $\xrightarrow{=}$, $\xrightarrow{+}$, $\xrightarrow{*}$, and $\leftrightarrow$.

Exercise 1.2 Let $\rightarrow$ be the symbolic differentiation relation introduced in the lecture.

(a) Compute the normal forms of the following terms.

  (i) $D_X(((X * X) * X) + (X * X))$

  (ii) $D_X((X * Y) + (Y * Y))$

(b) Prove that $\rightarrow$ is terminating.

Exercise 1.3 In the lecture, a group was defined by the following identities.

$$(x \circ y) \circ z \approx x \circ (y \circ z) \tag{G1}$$

$$e \circ x \approx x \tag{G2}$$

$$i(x) \circ x \approx e \tag{G3}$$

(a) Prove that groups satisfy the property that e is a right unit, i.e., prove that groups satisfy the following identity.

$$x \circ e \approx x \tag{G2'}$$

Hint. Show that $x \circ e$ can be transformed to x using the identities G1, G2, and G3.

(b) Consider the following identity.

$$x \circ i(x) \approx e \tag{G3'}$$

Prove that G1, G2, and G3' do not imply G2'.

Hint. Give a model of G1, G2, and G3' in which G2' does not hold; such a model exists with only two elements.

Exercise 1.4 Consider the following identities.

$$(x \circ y) \circ z \approx x \circ (y \circ z) \tag{R1}$$

$$(x \circ y) \circ x \approx x \tag{R2}$$

Prove or refute whether the following identities are implied by R1 and R2.

(a) $(x \circ x) \approx x$

(b) $(x \circ y) \circ z \approx x \circ z$