Exercise 1.1  Consider the reduction system \((M, \to)\) with
\[M := \{A_1, A_2, A_3, A_4, B_1, B_2, B_3, C_1, C_2, C_3, C_4, D, E\}\]
and \(\to \subseteq M \times M\) where
- \(A_1 \to B_1, A_1 \to B_2, A_2 \to B_1, A_2 \to B_2, A_3 \to B_3, A_4 \to B_3,\)
- \(B_1 \to C_1, B_2 \to C_2, B_2 \to C_3, B_3 \to C_1, B_3 \to C_2, B_3 \to C_3, B_3 \to C_4,\)
- \(C_3 \to E, C_4 \to E,\) and
- \(D \to C_4.\)

Answer the following questions.

(a) Which of the following properties are satisfied by \(\to\)? Justify your answer.
   (i) finite
   (ii) symmetric
   (iii) antisymmetric
   (iv) reflexive
   (v) irreflexive
   (vi) transitive

(b) Describe the closures \(\rightarrow, \leftarrow, \leftrightarrow,\) and \(\leftrightarrow.\)

Exercise 1.2  Let \(\to\) be the symbolic differentiation relation introduced in the lecture.

(a) Compute the normal forms of the following terms.
   (i) \(D_X(((X * X) * X) + (X * X))\)
   (ii) \(D_X((X * Y) + (Y * Y))\)

(b) Prove that \(\to\) is terminating.
**Exercise 1.3** In the lecture, a group was defined by the following identities.

\[
\begin{align*}
(x \circ y) \circ z &\approx x \circ (y \circ z) \quad \text{(G1)} \\
e \circ x &\approx x \\
i(x) \circ x &\approx e
\end{align*}
\]

(a) Prove that groups satisfy the property that \( e \) is a right unit, i.e., prove that groups satisfy the following identity.

\[
x \circ e \approx x
\]

\textit{Hint.} Show that \( x \circ e \) can be transformed to \( x \) using the identities G1, G2, and G3.

(b) Consider the following identity.

\[
x \circ i(x) \approx e
\]

Prove that G1, G2, and G3' do not imply G2'.

\textit{Hint.} Give a model of G1, G2, and G3' in which G2' does not hold; such a model exists with only two elements.

**Exercise 1.4** Consider the following identities.

\[
\begin{align*}
(x \circ y) \circ z &\approx x \circ (y \circ z) \\
(x \circ y) \circ x &\approx x
\end{align*}
\]

Prove or refute whether the following identities are implied by R1 and R2.

(a) \((x \circ x) \approx x\)

(b) \((x \circ y) \circ z \approx x \circ z\)