TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Term Rewriting Systems

Summer Semester 2018

Exercise Sheet 2 – Abstract Reduction Systems

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Exercise 2.1 Which of the closure operators defined in the lecture commute? Prove or refute the validity of each of the following equations.

(a) $(\xrightarrow{+})^{=} = (\xrightarrow{=})^{+}$

(b) $(\rightarrow \cup \xrightarrow{-1})^{+} = \xrightarrow{+} \cup (\xrightarrow{+})^{-1}$

(c) $(\xrightarrow{-1})^{+} = (\xrightarrow{+})^{-1}$

Exercise 2.2 Consider the following reduction relations.

- Let M be a set. We define the reduction relation $\rightarrow_M$ on the power set $\wp(M)$ as follows.

$$A \rightarrow_M B \quad \text{if} \quad B \subsetneq A$$

- For natural numbers $p, q \in \mathbb{N}$ we define the reduction relation $\rightarrow_{p,q}$ on $\mathbb{N}$ as follows.

$$\begin{aligned} & n \rightarrow_{p,q} n - p \quad \text{if} \quad n > p \\ \text{and} \quad & n \rightarrow_{p,q} n - q \quad \text{if} \quad n > q \end{aligned}$$

For each of the following properties, describe those sets M and those natural numbers p, q such that $\rightarrow_M$ and $\rightarrow_{p,q}$ satisfy this property.

(a) terminating

(b) Church-Rosser

(c) normalising

(d) confluent

Exercise 2.3 Disprove the following claim.

If $(M, \rightarrow)$ is a reduction system such that $\rightarrow$ is decidable, then the set $\{ x \in M \mid x \text{ is reducible} \}$ is also decidable.

Exercise 2.4 Consider the reduction system $(\{a,b\}^*, \to_S)$ from the lecture where

$$\to_S = \{\, (uabbv, uaav) \mid u,v \in \{a,b\}^* \,\} \cup \{\, (uav, ubv) \mid u,v \in \{a,b\}^* \,\}.$$

(a) Decide whether the following statements hold true.

(i) $ababb \overset{*}{\to}_S bbb$

(ii) $aabb \overset{*}{\leftrightarrow}_S aaaaa$

(iii) $abababbbab \overset{*}{\leftrightarrow}_S aaaaaaaaababababababbababababbababababbababababaaaaaaaaaabb$

(b) Prove that $\to_S$ is finitely branching.

(c) Prove that $\to_S$ is terminating.

Hint. Define a monotone mapping $\phi\colon (\{a,b\}^*, \to_S) \to (\mathbb{N}, >)$, i.e., a mapping such that $u \to_S v$ implies $\phi(u) > \phi(v)$ for all $u, v \in \{a,b\}^*$.