TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Term Rewriting Systems

Summer Semester 2018

Exercise Sheet 3 – Abstract Reduction Systems

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Exercise 3.1 As proven in the lecture, a finitely branching relation terminates if, and only if, there is a monotone embedding into $(\mathbb{N}, >)$. Show that the restriction to finitely branching relations is necessary.

In particular, let the reduction system $(\mathbb{N} \times \mathbb{N}, \to)$ be defined by the two rules

$$(i+1, j) \to (i, k) \quad \text{and} \quad (i, j+1) \to (i, j)$$

for all $i, j, k \in \mathbb{N}$, and prove that it is not finitely branching, it is terminating, and there does not exist any monotone embedding into $(\mathbb{N}, >)$.

Exercise 3.2 Although for each finitely branching, terminating reduction, there must exist a monotone embedding into $(\mathbb{N}, >)$, it can be tricky to find one.

To clarify this further, we consider the reduction system $(\mathbb{N} \times \mathbb{N}, \to)$ with the rules

$$(i, j+1) \to (i, j) \quad \text{and} \quad (i+1, j) \to (i, i)$$

for all $i, j \in \mathbb{N}$. Demonstrate that it is finitely branching and terminating, and find a monotone embedding into $(\mathbb{N}, >)$.

Exercise 3.3 A relation $\to$ is called *bounded* if for each element, the length of all paths starting from it is bounded, that is, for each x, there is some n such that for all $x \xrightarrow{k} y$, it holds true that $k \leq n$.

(a) Is every terminating relation bounded?

(b) Show that a finitely branching relation terminates if, and only if, it is bounded.

Exercise 3.4 Show that $\xrightarrow{+}$ is terminating if, and only if, $\to$ is terminating.

Exercise 3.5 Prove or refute each of the following statements.

(a) Any acyclic relation is terminating if it is globally finite.

(b) A finitely branching and acyclic relation is globally finite if, and only if, it is terminating.