TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Term Rewriting Systems

Summer Semester 2018

Exercise Sheet 6 – Equational Problems

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Exercise 6.1 Give a trivial example of a finite set E of identities such that the ground word problem for E is decidable but the word problem is not.

Exercise 6.2 Show that the word problem for E is decidable if all equivalence classes of $\approx_E$ are finite.

Exercise 6.3 Show that $\approx_D$, equality modulo distributivity, is decidable.

$$D \coloneqq \{x * (y + z) \approx (x * y) + (x * z),\ (x + y) * z \approx (x * z) + (y * z)\}$$

Exercise 6.4 Let R be a term rewriting system. Show the following statements.

(a) $\to_R$ is the least rewrite relation containing R.

(b) $\overset{*}{\to}_R$, $\overset{+}{\to}_R$, and $\overset{*}{\leftrightarrow}_R$ are rewrite relations.

Exercise 6.5 Let $G_1 \coloneqq \{f^3(a) \approx a,\ f^5(a) \approx a\}$ and $G_2 \coloneqq \{f^4(a) \approx a,\ f^6(a) \approx a\}$. Consider the congruence closures of G_1 and G_2 and verify whether $f^2(a) \approx a$ holds true in G_1 or G_2.

Exercise 6.6 Let u, x, y, z be variables. Use the unification algorithm on Page 74 in *Term Rewriting and All That* to solve the following two unification problems.

(a) $S_1 \coloneqq \{f(h(x), g(x,u)) \stackrel{?}{=} f(z, g(f(y,y),z))\}$

(b) $S_2 \coloneqq \{h(x, g(x,y), y) \stackrel{?}{=} h(x, g(a,y), y),\ z \stackrel{?}{=} h(x, g(x,b), b)\}$

Exercise 6.7 From the unification algorithm on Page 74 in *Term Rewriting and All That*, design a direct decision procedure for the matching problem.

Hint. "Direct" means that no constants are introduced in the right term. Instead, the rules are to be modified such that the new algorithm returns either "the input terms do not match" or a matcher for the input terms as soon as possible.