TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Term Rewriting Systems

Summer Semester 2018

Exercise Sheet 7 – Termination

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Prof. Dr.-Ing. Franz Baader, Dipl.-Math. Francesco Kriegel

Exercise 7.1 Consider the following modification of the reduction described in Subsection 5.1.1.

For a given Turing machine $\mathcal{M}$, let

$$\Sigma'_{\mathcal{M}} \coloneqq \{s_0, \ldots, s_n\} \cup \{q_0, \ldots, q_p\} \cup \{\overrightarrow{\ell}, \overleftarrow{r}\},$$

and let $R'_{\mathcal{M}}$ be the rewrite system that is obtained from $R_{\mathcal{M}}$ be replacing both $\overrightarrow{s_i}$ and $\overleftarrow{s_i}$ by s_i. Give an example of a terminating Turing machine $\mathcal{M}$ for which $R'_{\mathcal{M}}$ is not terminating.

Exercise 7.2 Show that the following is not a decision procedure for termination of a ground term rewriting system $R \coloneqq \{\ell_1 \to r_1, \ldots, \ell_n \to r_n\}$.

> Generate all reduction sequences starting with r_1. If one of these sequences yields a term that has r_1 as subterm, then answer "R is not terminating". Otherwise, continue with r_2, etc. Eventually, answer "R is terminating".

Exercise 7.3 Prove the analogue of Theorem 5.1.9 for left-ground systems, and explain why this is not an interesting generalization of the theorem for the ground case.

Exercise 7.4 A term rewriting system R is called *right-reduced* if for all $\ell \to r \in R$, it holds true that r is R-irreducible. Show that a right-ground term rewriting system is right-reduced only if it is terminating.

Exercise 7.5 We define a relation $>$ on $T(\Sigma, V)$ by

$$s > t \text{ if } |s| > |t| \text{ and } |s|_x \geq |t|_x \text{ for all } x \in V.$$

Demonstrate that $>$ is a reduction order.