Term Rewriting Systems

Exercise Sheet 7 – Termination

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Exercise 7.1  Prove undecidability of the *uniform halting problem*.

*Hint.*

Exercise 7.2  Consider the following modification of the reduction described in Subsection 5.1.1.

For a given Turing machine $\mathcal{M}$, let

$$\Sigma'_\mathcal{M} := \{s_0, \ldots, s_n\} \cup \{q_0, \ldots, q_p\} \cup \{\ell, \rightarrow, \leftarrow\},$$

and let $R'_\mathcal{M}$ be the rewrite system that is obtained from $R\mathcal{M}$ by replacing both $s_i$ and $s_i$ by $s_j$. Give an example of a terminating Turing machine $\mathcal{M}$ for which $R'_\mathcal{M}$ is not terminating.

Exercise 7.3  Show that the following is not a decision procedure for termination of a ground term rewriting system $R := \{\ell_1 \rightarrow r_1, \ldots, \ell_n \rightarrow r_n\}$.

Generate all reduction sequences starting with $r_1$. If one of these sequences yields a term that has $r_1$ as subterm, then answer “$R$ is not terminating”. Otherwise, continue with $r_2$, etc. Eventually, answer “$R$ is terminating”.

Exercise 7.4  Prove the analogue of Theorem 5.1.9 for left-ground systems, and explain why this is not an interesting generalization of the theorem for the ground case.

Exercise 7.5  A term rewriting system $R$ is called *right-reduced* if for all $\ell \rightarrow r \in R$, it holds true that $r$ is $R$-irreducible. Show that a right-ground term rewriting system is right-reduced only if it is terminating.

Exercise 7.6  We define a relation $>$ on $T(\Sigma, V)$ by

$$s > t \text{ if } |s| > |t| \text{ and } |s|_x \geq |t|_x \text{ for all } x \in V.$$ 

Demonstrate that $>$ is a reduction order.