TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Term Rewriting Systems

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Exercise Sheet 8 – Termination

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Exercise 8.1 (a) Let $P \in \mathbb{Z}[X]$ where $P = \sum_{i=0}^{n} c_i \cdot X^i$ be a polynomial with one indeterminate X and coefficients c_i in $\mathbb{Z}$. Prove for all $a \in \mathbb{Z}$: if $P(a) = 0$, then $a \mid c_0$, i.e., any root of P divides c_0.

(b) Devise a decision procedure that, for each polynomial $P \in \mathbb{Z}[X]$, decides if P has a root in $\mathbb{Z}$.

(c) Show that for polynomials with more than one indeterminate the roots need not satisfy such a property.

Hint. A *root* of a polynomial $P \in \mathbb{Z}[X_1, \ldots, X_n]$ is a tuple $(a_1, \ldots, a_n) \in \mathbb{Z}^n$ such that $P(a_1, \ldots, a_n) = 0$.

Exercise 8.2 Show that undecidability of Hilbert's 10th Problem implies that the following problem is undecidable.

Instance: Two polynomials $P, Q \in \mathbb{N}[X_1, \ldots, X_n]$ in n indeterminates with non-negative integer coefficients, and a (decidable) subset A of $\mathbb{N}$.

Question: Does $P >_A Q$ hold, i.e., is the value of P greater than the value of Q for all valuations with elements in A.

Show that this implies that there exists a polynomial interpretation $\mathcal{A}$ for which it is in general undecidable whether two terms ℓ and r satisfy $\ell >_{\mathcal{A}} r$.

Exercise 8.3 Let R be a finite term rewriting system and f be a function symbol. For a term t, let $|t|_f$ denote the number of occurences of f in t. Show that there exists a positive integer k such that $s \to_R t$ implies $|t|_f \leq k \cdot (|s|_f + 1)$ for all terms s and t.

Exercise 8.4 Use the previous exercise and the fact that Ackermann's function is growing faster than any primitive recursive function to show that the length of reduction sequences for the term rewriting system R_{Ack} of Example 5.3.11 in *Term Rewriting and All That* cannot be bounded by a primitive recursive function.

Exercise 8.5 Consider the term rewriting system

$$R := \{g(x, g(y, z)) \to g(g(x, y), z),\ g(g(x, y), z) \to g(y, y)\}.$$

Use a polynomial interpretation to prove that R terminates.

Hint. Try $P_g\colon (X, Y) \mapsto X \cdot Y + Y^2$.

Exercise 8.6 Consider the term rewriting system $R := \{f(f(x)) \to f(g(f(x)))\}$ from Example 5.4.9 in *Term Rewriting and All That*. It was shown that its termination cannot be proven using a simplification order.

(a) Find an alternative way to prove that R is terminating.

Hint. Use the interpretation method.

(b) Is there a polynomial order that can be used to prove termination of R?

Exercise 8.7 Let $\mathcal{A}$ be a monotone polynomial interpretation of some signature Σ. Prove that the polynomial order $>_{\mathcal{A}}$ is a simplification order if the following properties are satisfied.

- Σ contains only function symbols of arity at least 2.
- The carrier set of $\mathcal{A}$ does not contain 1, i.e., $A \subseteq \mathbb{N} \setminus \{0, 1\}$.

Are those conditions necessary?