



Term Rewriting Systems

Summer Semester 2018

Exercise Sheet 10 – Confluence

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Exercise 4 will not be discussed during the tutorial. Instead, solutions can be handed in at the next tutorial or, alternatively, sent to `francesco.kriegel@tu-dresden.de`. Individual feedback will then be provided.

Exercise 10.1 Find terms r_1 and r_2 such that $\{f(g(x)) \rightarrow r_1, g(h(x)) \rightarrow r_2\}$ is confluent.

Exercise 10.2 Is $R := \{f(g(f(x))) \rightarrow g(x)\}$ confluent? Find a convergent R' such that $\approx_R = \approx_{R'}$.

Exercise 10.3 Compute all critical pairs for the term rewriting system R consisting of the following rules. Is R locally confluent? Is it convergent?

$$\begin{aligned} 0 + y &\rightarrow y \\ x + 0 &\rightarrow x \\ s(x) + y &\rightarrow s(x + y) \\ x + s(y) &\rightarrow s(x + y) \end{aligned}$$

★ **Exercise 10.4** In Section 6 in *Term Rewriting and All That*, the proof of local confluence in Case 2.1 relies entirely on pictures. For an algebraic proof define the substitution σ'_1 by $\sigma'_1 x := (\sigma_1 x)[\sigma_2 r_2]_{q_2}$ and $\sigma'_1 y := \sigma_1 y$ for all $y \neq x$. Using the lemma below (which you should also prove) show formally that $\sigma_1 r_1 \xrightarrow{*} \sigma'_1 r_1 \xleftarrow{*} (\sigma_1 \ell_1)[\sigma_2 \ell_2]_{q_1}$.

Lemma. Let x be some fixed variable and let σ and σ' be substitutions such that $\sigma x \rightarrow \sigma' x$ and $\sigma y = \sigma' y$ for all $y \neq x$. If $\{o_1, \dots, o_n\}$ is the set of positions in some term t such that $t|_{o_i} = x$, then $t_i \xrightarrow{n-i} \sigma' t$, where $t_0 := \sigma t$ and $t_{i+1} := t_i[\sigma' x]_{o_i}$.

Exercise 10.5 What is the difficulty if you try to put a complexity bound in terms of the size of R on the running time of the decision procedure outlined in the proof of Corollary 6.2.6 in *Term Rewriting and All That*?

Exercise 10.6 Consider the system $\{f(x) \rightarrow g(x, y)\}$. Does it have any critical pairs? Is the induced rewrite relation confluent? What is going wrong here?

Exercise 10.7 Show that the term rewriting system R consisting of the following rules is confluent.

$$(x * y) * (y * z) \rightarrow y$$

$$x * ((x * y) * z) \rightarrow x * y$$

$$(x * (y * z)) * z \rightarrow y * z$$