



## Term Rewriting Systems

Summer Semester 2018

### Exercise Sheet 11 – Completion

27th June 2018

Prof. Dr.-Ing. Franz Baader, Dipl.-Math. Francesco Kriegel

**Exercise 11.1** Consider the following sets of identities.

$$E_1 := \{f(g(f(x))) \approx x\}$$

$$E_2 := \{f(g(f(x))) \approx f(g(x))\}$$

- (a) Choose an appropriate reduction order  $>$  and apply the basic completion procedure to the inputs  $(E_1, >)$  and  $(E_2, >)$ .
- (b) What happens if the improved completion procedure that also simplifies rules is applied to  $E_1$ ?

**Exercise 11.2** Show that the encompassment relation  $\sqsupseteq$  is a quasi-order, and that its strict part  $\sqsubset$  is a well-founded strict partial order.

**Exercise 11.3** Let  $\equiv$  denote the equivalence relation that is induced by  $\sqsupseteq$ , i.e.,  $s \equiv t$  if, and only if,  $s \sqsupseteq t$  and  $t \sqsupseteq s$ . Show the following statements.

- (a) It holds true that  $s \equiv t$  if, and only if,  $s$  and  $t$  are equal up to renaming of variables.
- (b) For a given term  $s$ , there exists up to  $\equiv$  only finitely many terms  $t$  such that  $s \sqsupseteq t$ .