TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Term Rewriting Systems

Summer Semester 2018

Exercise Sheet 12 – Completion

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Exercise 12.1 (a) Apply the rules of the improved completion procedure to the following set of identities.

$$E \coloneqq \{f(f(x,y),z) \approx f(x,f(y,z)),\ f(x,x) \approx x,\ f(f(x,y),x) \approx x\}$$

Use a strategy that resembles the basic completion procedure, but simplifies rules as follows: upon adding new rules, simplify old ones by means of L-Simplify-rule and R-Simplify-rule. For the proof

$$P \coloneqq \langle f(x,f(y,f(y,x))),\ f(x,f(f(y,y),x)),\ f(x,f(y,x)),\ f(f(x,y),x),\ x\rangle$$

construct a rewrite proof P' in R_ω with $P \succ_{\mathcal{C}} P'$ using the proof of Lemma 7.3.4 in *Term Rewriting and All That*.

(b) Fix the following set of identities.

$$E \coloneqq \{x + (y+z) \approx (x+y) + z,\ f(x) + f(y) \approx f(x+y)\}$$

Apply the completion procedure described above to the input E and the polynomial order induced by

$$P_f(X) \coloneqq X + 1 \quad \text{and} \quad P_+(X,Y) \coloneqq X \cdot Y^2.$$

Exercise 12.2 Consider the following completion procedure for ground term rewriting systems.

Input: A finite set G_0 of ground identities over Σ and a reduction order $>$ that is total on the set of ground terms over Σ.

Procedure: Exhaustively apply the rules L-Simplify-rule, Delete, and Orient.

Output: A ground term rewriting system.

Show that this procedure

(a) always terminates,

(b) is fair,

(c) is correct, and

(d) never fails.

Exercise 12.3 The simple semi-decision procedure described in the proof of Theorem 7.3.5 in *Term Rewriting and All That* is rather inefficient, because after each inference step it tests whether s and t are joinable by computing *all* normal forms of s and t with respect to the (possibly non-confluent) term rewrite system R_n.

Show that the following modification of the procedure still yields a semi-decision procedure for the word problem.

- Begin with $s_0 := s$ and $t_0 := t$.
- After the ith inference step, compute *one arbitrary* R_i-normal form s_i of s_{i-1} and *one arbitrary* R_i-normal form t_i of t_{i-1}.
- Output "yes" if there exists some n such that $s_n = t_n$.

Hint. Show the following.

(a) $s \overset{*}{\leftrightarrow}_{E_0} s_n$ and $t \overset{*}{\leftrightarrow}_{E_0} t_n$ for all $n \geq 0$.

(b) Since R_∞ is terminating, there is an $n \geq 0$ such that $s_n = s_m$ and $t_n = t_m$ for all $m \geq n$.