TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Description Logic

Summer Semester 2019

Exercise Sheet 4

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Exercise 4.1 Prove or refute the following claim: If an $\mathcal{ALC}$-concept description C is satisfiable w.r.t. an $\mathcal{ALC}$-TBox $\mathcal{T}$, then for all $n \geq 1$ there is a finite model $\mathcal{I}_n$ of $\mathcal{T}$ such that $|C^{\mathcal{I}_n}| \geq n$.

Does the claim hold true if the condition "$|C^{\mathcal{I}_n}| \geq n$" is replaced by "$|C^{\mathcal{I}_n}| = n$"?

Exercise 4.2 Prove or refute the following claim: Given an $\mathcal{ALC}$-concept description C and an $\mathcal{ALC}$-TBox $\mathcal{T}$, if $\mathcal{I}$ is an interpretation and $\mathcal{J}$ its filtration w.r.t. $\mathsf{sub}(C) \cup \mathsf{sub}(\mathcal{T})$ (see Definition 3.14), then the relation $\rho := \{(d, [d]) \mid d \in \Delta^{\mathcal{I}}\}$ is a bisimulation between $\mathcal{I}$ and $\mathcal{J}$.

Hint. If the above relation ρ were a bisimulation, why do we have to explicitly prove Lemma 3.15 in the lecture? Wouldn't Lemma 3.15 then be a consequence of Theorem 3.2?

Exercise 4.3 We consider bisimulations between an interpretation $\mathcal{I}$ and itself, which are called bisimulations *on* $\mathcal{I}$. For two elements $d, e \in \Delta^{\mathcal{I}}$, we write $d \approx_{\mathcal{I}} e$ if they are *bisimilar*, i.e., if there is a bisimulation ρ on $\mathcal{I}$ such that $d \; \rho \; e$.

(a) Show that $\approx_{\mathcal{I}}$ is an equivalence relation on $\Delta^{\mathcal{I}}$.

(b) Show that $\approx_{\mathcal{I}}$ is a bisimulation on $\mathcal{I}$.

(c) Show that, for finite interpretations $\mathcal{I}$, the relation $\approx_{\mathcal{I}}$ can be computed in time polynomial in the cardinality of $\mathcal{I}$ (if the signature is finite as well).

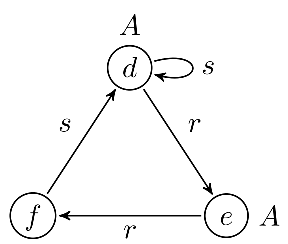
Consider the interpretation $\mathcal{J}$ that is defined like the filtration (Definition 3.14), but with $\approx_{\mathcal{I}}$ instead of $\simeq$.

(d) Show that $\rho := \{(d, [d]_{\approx_{\mathcal{I}}}) \mid d \in \Delta^{\mathcal{I}}\}$ is a bisimulation between $\mathcal{I}$ and $\mathcal{J}$.

(e) Show that, if $\mathcal{I}$ is a model of an $\mathcal{ALC}$-concept description C w.r.t. an $\mathcal{ALC}$-TBox $\mathcal{T}$, then so is $\mathcal{J}$.

(f) Why can we not use the previous result to show the finite model property for $\mathcal{ALC}$?

Exercise 4.4 For the following interpretation $\mathcal{I}$, draw the unraveling of $\mathcal{I}$ at d up to depth 5, i.e., restricted to d-paths of length at most 5 (see Definition 3.21):



Exercise 4.5 Prove or refute the following claim: If $\mathcal{K}$ is an $\mathcal{ALC}$-knowledge base and C is an $\mathcal{ALC}$-concept description such that C is satisfiable w.r.t. $\mathcal{K}$, then C has a tree model w.r.t. $\mathcal{K}$.