TECHNISCHE UNIVERSITÄT DRESDEN

**Faculty of Computer Science** Institute of Theoretical Computer Science, Chair of Automata Theory

# Description Logic

Summer Semester 2019

## Exercise Sheet 5

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**Exercise 5.1** Prove that the negation normal form (NNF) of an $\mathcal{ALC}$ concept description $C$ can be computed in polynomial time and is equivalent to $C$.

**Exercise 5.2** Execute the tableau algorithm $\mathsf{consistent}(\mathcal{A})$ for the normalized ABox

$$\mathcal{A} \coloneqq \{(b,a):r,\ (a,b):r,\ (a,c):s,\ (c,b):s,\ a:\exists s.A,\ b:\forall r.\big((\forall s.\neg A)\sqcup(\exists r.B)\big),\ c:\forall s.\big(B\sqcap(\forall s.\bot)\big)\}.$$

If $\mathcal{A}$ is consistent, draw the model generated by the algorithm.

**Exercise 5.3** We consider the concept constructor $\rightarrow$ (implication) with the following semantics:

$$(C\rightarrow D)^{\mathcal{I}} \coloneqq \{x\in\Delta^{\mathcal{I}} \mid x\in C^{\mathcal{I}} \text{ implies } x\in D^{\mathcal{I}}\}.$$

To extend $\mathsf{consistent}(\mathcal{A})$ to this constructor, we propose two alternative new expansion rules:

| The deterministic $\rightarrow$-rule | |
|---|---|
| *Condition:* | $\mathcal{A}$ contains $a:C\rightarrow D$ and $a:C$, but not $a:D$ |
| *Action:* | $\mathcal{A}\longrightarrow\mathcal{A}\cup\{a:D\}$ |

| The nondeterministic $\rightarrow$-rule | |
|---|---|
| *Condition:* | $\mathcal{A}$ contains $a:C\rightarrow D$, but neither $a:\dot\neg C$ nor $a:D$ |
| *Action:* | $\mathcal{A}\longrightarrow\mathcal{A}\cup\{a:X\}$ for some $X\in\{\dot\neg C, D\}$ |

For each rule, determine whether the resulting algorithm is still terminating, sound, and complete.

**Exercise 5.4** We consider TBoxes that only contain the following two kinds of axioms:

- role inclusions of the form $r\sqsubseteq s$, and
- role disjointness constraints of the form $\mathsf{disj}(r,s)$,

where $r$ and $s$ are role names. An interpretation $\mathcal{I}$ satisfies these axioms if

- $r^{\mathcal{I}}\subseteq s^{\mathcal{I}}$, and
- $r^{\mathcal{I}}\cap s^{\mathcal{I}}=\emptyset$, respectively.

Modify the tableau algorithm $\mathsf{consistent}(\mathcal{A})$ to decide consistency of $(\mathcal{T},\mathcal{A})$, where $\mathcal{A}$ is an ABox and $\mathcal{T}$ a TBox that contains only role inclusions and role disjointness constraints. Show that the algorithm remains terminating, sound, and complete.