Description Logic

Summer Semester 2019

Exercise Sheet 8

21st May 2019

Prof. Dr.-Ing. Franz Baader, Dipl.-Math. Francesco Kriegel

Exercise 8.1 Prove that the tableau algorithm for $\mathcal{ALCN}$ is sound and complete.

Exercise 8.2 We extend the tableau algorithm from $\mathcal{ALCN}$ to $\mathcal{ALCQ}$ by modifying the $\geq$-rule and the $\leq$-rule as follows:

The $\geq$-rule	
Condition:	$\mathcal{A}$ contains $a:(\geq n\,r.C)$, but there are no n distinct individuals $b_1, \ldots, b_n$ with $\{(a, b_i):r,\ b_i:C \mid 1 \leq i \leq n\} \subseteq \mathcal{A}$, and a is not blocked
Action:	$\mathcal{A} \longrightarrow \mathcal{A} \cup \{(a, d_i):r,\ d_i:C \mid 1 \leq i \leq n\} \cup \{d_i \neq d_j \mid 1 \leq i < j \leq n\}$, where $d_1, \ldots, d_n$ are new individual names

The $\leq$-rule	
Condition:	$\mathcal{A}$ contains $a:(\leq n\,r.C)$, and there are $n+1$ distinct individuals $b_0, \ldots, b_n$ with $\{(a, b_i):r,\ b_i:C \mid 0 \leq i \leq n\} \subseteq \mathcal{A}$
Action:	$\mathcal{A} \longrightarrow \mathsf{prune}(\mathcal{A}, b_j)[b_j \mapsto b_i] \cup \{b_i = b_j\}$ for $i \neq j$ such that, if b_j is a root individual, then so is b_i

For the knowledge base

$$\big(\{C \sqsubseteq E\},\ \{a:(\leq 1\,r.(D \sqcap E)),\ (a,b):r,\ b:C \sqcap D,\ (a,c):r,\ c:D \sqcap E,\ c:\neg C\}\big),$$

determine whether it is consistent, and whether the proposed algorithm detects this.

Exercise 8.3 Let $\mathcal{T}$ be an acyclic TBox in NNF, and let $\mathcal{T}^{\sqsubseteq}$ be obtained from $\mathcal{T}$ by replacing each concept definition $A \equiv C$ with the concept inclusion $A \sqsubseteq C$.

Prove that every concept name is satisfiable w.r.t. $\mathcal{T}$ if, and only if, it is satisfiable w.r.t. $\mathcal{T}^{\sqsubseteq}$. Does this also hold true for the acyclic TBox $\{A \equiv C \sqcap \neg B,\ B \equiv P,\ C \equiv P\}$?

Exercise 8.4 Use the $\mathcal{ALC}$-Worlds algorithm to decide satisfiability of the concept name A_0 w.r.t. the following simple TBox:

$$\begin{aligned}
\{\ & A_0 \equiv A_1 \sqcap A_2, && A_1 \equiv \exists r.A_3, && A_2 \equiv A_4 \sqcap A_5, && A_3 \equiv P, \\
& A_4 \equiv \exists r.A_6, && A_5 \equiv A_7 \sqcap A_8, && A_6 \equiv Q, && A_7 \equiv \forall r.A_4, \\
& A_8 \equiv \forall r.A_9, && A_9 \equiv \forall r.A_{10}, && A_{10} \equiv \neg P\ \}
\end{aligned}$$

Draw the recursion tree of a successful run and of an unsuccessful run. Does the algorithm return a positive or a negative result on this input?