TECHNISCHE UNIVERSITÄT DRESDEN

**Faculty of Computer Science** Institute of Theoretical Computer Science, Chair of Automata Theory

# Description Logic

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## Exercise Sheet 9

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**Exercise 9.1** Determine whether Player $1$ has a winning strategy in the following finite Boolean games, where in both cases $\Gamma_1 := \{p_1, p_3\}$ and $\Gamma_2 := \{p_2, p_4\}$.

(a) $((p_1 \wedge p_3) \rightarrow \neg p_2) \wedge (\neg p_1 \rightarrow p_1) \wedge (\neg p_2 \rightarrow (p_3 \vee p_4))$

(b) $(p_1 \vee \neg p_2) \wedge (p_2 \vee p_3) \wedge (\neg p_3 \vee \neg p_4) \wedge (\neg p_1 \vee \neg p_2 \vee p_3 \vee p_4)$

**Exercise 9.2** Use the $\mathcal{ALC}$-Elim algorithm to decide satisfiability of

(a) the concept name $A$ w.r.t. $\mathcal{T} := \{A \sqsubseteq \exists r.A,\ \top \sqsubseteq A,\ \forall r.A \sqsubseteq \exists r.A\}$, and

(b) the concept description $\forall r.\forall r.\neg B$ w.r.t. $\mathcal{T} := \{\neg A \sqsubseteq B,\ A \sqsubseteq \neg B,\ \top \sqsubseteq \neg \forall r.A\}$.

Give the constructed type sequence $\Gamma_0, \Gamma_1, \ldots$. In case of satisfiability, also give the satisfying model constructed in the proof of Lemma 5.10.

**Exercise 9.3** Extend the $\mathcal{ALC}$-Elim algorithm to the description logic $\mathcal{ALCI}$. Prove the correctness of the extended algorithm.

**Exercise 9.4** The description logic $\mathcal{S}$ extends $\mathcal{ALC}$ with *transitivity axioms* $\mathsf{trans}(r)$ for role names $r \in \mathbf{R}$. Their semantics is defined as follows: $\mathcal{I} \models \mathsf{trans}(r)$ if, and only if, $r^{\mathcal{I}}$ is transitive. Furthermore, an $\mathcal{S}$ knowledge base $(\mathcal{T}, \mathcal{A}, \mathcal{R})$ consists of an $\mathcal{ALC}$ knowledge base $(\mathcal{T}, \mathcal{A})$ and an *RBox* $\mathcal{R}$ of transitivity axioms. Prove the following claims:

(a) $\mathsf{trans}(r)$ cannot be expressed in $\mathcal{ALC}$, i.e., $\mathcal{S}$ is more expressive than $\mathcal{ALC}$.
*Hint.* It is possible to show that $\forall x.\forall y.\forall z.(R(x,y) \wedge R(y,z)) \rightarrow R(x,z)$ is not equivalent to a formula in the two variable fragment of FOL, which has the finite model property.

(b) For an arbitrary TBox $\mathcal{T}$, the concept description $C_{\mathcal{T}}$ is defined as $\bigsqcap_{C \sqsubseteq D \in \mathcal{T}} \neg C \sqcup D$. Then $\mathcal{T}$ and $\{\top \sqsubseteq C_{\mathcal{T}}\}$ have the same models.

(c) Let $\mathcal{K} = (\mathcal{T}, \mathcal{A}, \mathcal{R})$ be an $\mathcal{S}$ knowledge base such that w.l.o.g. $\mathcal{T}$ consists of a single GCI $\top \sqsubseteq C_{\mathcal{T}}$, and $C_{\mathcal{T}}$ is in NNF. Define the $\mathcal{ALC}$ knowledge base $\mathcal{K}^+ := (\mathcal{T}^+, \mathcal{A})$ where

$$\mathcal{T}^+ := \mathcal{T} \cup \{\, \forall r.C \sqsubseteq \forall r.\forall r.C \mid \mathsf{trans}(r) \in \mathcal{R} \text{ and } \forall r.C \in \mathsf{Sub}(C_{\mathcal{T}}) \,\}.$$

Then $\mathcal{K}$ is consistent if, and only if, $\mathcal{K}^+$ is consistent. Consequently, the tableaux algorithm for $\mathcal{ALC}$ can also be utilized for $\mathcal{S}$.

(d) The problem of deciding consistency of an $\mathcal{S}$ knowledge base (with a general TBox) is ExpTime-complete.