# Description Logic

Summer Semester 2019

## Exercise Sheet 10

5th June 2019

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**Exercise 10.1** Determine whether Player $2$ has a winning strategy in the following infinite Boolean games where the initial configuration $t_0$ assigns *false* to all variables.

(a) $\varphi \coloneqq (p_1 \wedge p_2 \wedge \neg q_1) \vee (p_3 \wedge p_4 \wedge \neg q_2) \vee (\neg(p_1 \vee p_4) \wedge q_1 \wedge q_2)$
with $\Gamma_1 \coloneqq \{p_1, p_2, p_3, p_4\}$ and $\Gamma_2 \coloneqq \{q_1, q_2\}$

(b) $\varphi \coloneqq \big((p_1 \leftrightarrow \neg q_1) \wedge (p_2 \leftrightarrow \neg q_2) \wedge (p_1 \leftrightarrow p_2)\big) \vee \big((p_1 \leftrightarrow q_1) \wedge (p_2 \leftrightarrow q_2) \wedge (p_1 \leftrightarrow \neg p_2)\big)$
with $\Gamma_1 \coloneqq \{p_1, p_2\}$ and $\Gamma_2 \coloneqq \{q_1, q_2\}$

**Exercise 10.2** Are the following variations of infinite Boolean games also ExpTime-hard?

(a) Player $1$ wins if the constructed truth assignment falsifies the formula $\varphi$ instead of satisfying it.

(b) Player $2$ starts instead of Player $1$.

(c) The variables are not assigned to a specific player; instead, the active player can choose any variable and assign to it a new truth value; variables can be chosen multiple times.

(d) The two players must always flip the assignment of a variable, i.e., the truth assignment cannot be left unchanged.

**Exercise 10.3** The *universal role* is a role name $u$ such that its extension is fixed as $\Delta^\mathcal{I} \times \Delta^\mathcal{I}$ in any interpretation $\mathcal{I}$. Let $\mathcal{ALC}^u$ denote the extension of $\mathcal{ALC}$ with the universal role. Show that concept satisfiability in $\mathcal{ALC}^u$ without TBoxes is ExpTime-complete.

**Exercise 10.4** A *role complement* is a role of the form $\neg r$ where $r$ is a role name. The semantics of role complements is defined as follows.

$$(\neg r)^\mathcal{I} \coloneqq \Delta^\mathcal{I} \times \Delta^\mathcal{I} \setminus r^\mathcal{I}$$

The description logic $\mathcal{ALC}^\neg$ extends $\mathcal{ALC}$ by role complements, i.e., role complements are allowed to occur in existential restrictions, value restrictions, and role assertions. Show that $\mathcal{ALC}^\neg$ does not have the tree model property.