Description Logic

Summer Semester 2019

Exercise Sheet 11

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Exercise 11.1 We consider the extension of $\mathcal{ALC}$ with feature names. Fix some subset $\mathbf{F} \subseteq \mathbf{R}$. The elements of $\mathbf{F}$ are called *feature names*. Of course, each feature name f is also a role name and must thus be interpreted as a binary relation over $\Delta^{\mathcal{I}}$ for each interpretation $\mathcal{I}$. However, it is further required that the extension $f^{\mathcal{I}}$ is functional, i.e., $f^{\mathcal{I}}$ can be treated as a partial function on $\Delta^{\mathcal{I}}$.

Let $f_1, \ldots, f_m$ and $g_1, \ldots, g_n$ be (not necessarily distinct) feature names. A *feature agreement* is a concept of the form $(f_1 \circ \cdots \circ f_m) \downarrow (g_1 \circ \cdots \circ g_n)$ with the following semantics.

$$\big((f_1 \circ \cdots \circ f_m) \downarrow (g_1 \circ \cdots \circ g_n)\big)^{\mathcal{I}} \coloneqq \{\, \delta \mid \delta \in \Delta^{\mathcal{I}} \text{ and } (f_1^{\mathcal{I}} \circ \cdots \circ f_m^{\mathcal{I}})(\delta) = (g_1^{\mathcal{I}} \circ \cdots \circ g_n^{\mathcal{I}})(\delta) \,\}$$

Feature disagreements $(f_1 \circ \cdots \circ f_m) \uparrow (g_1 \circ \cdots \circ g_n)$ are defined analogously. The description logic $\mathcal{ALCF}$ extends $\mathcal{ALC}$ with feature agreements and feature disagreements.

Show that satisfiability w.r.t. general TBoxes is undecidable for $\mathcal{ALCF}$.

Exercise 11.2 Fix two concept names A and B as well as some role name r. Does the entailment $\{\forall r.A \sqsubseteq \exists r.A\} \models \forall r.B \sqsubseteq \exists r.B$ hold true?

Exercise 11.3 We consider simulations, which are "one-sided" variants of bisimulations. Given interpretations $\mathcal{I}$ and $\mathcal{J}$, the relation $\sigma \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{J}}$ is a *simulation* from $\mathcal{I}$ to $\mathcal{J}$ if

- whenever $\delta \;\sigma\; \epsilon$ and $\delta \in A^{\mathcal{I}}$, then $\epsilon \in A^{\mathcal{J}}$, for all $\delta \in \Delta^{\mathcal{I}}$, $\epsilon \in \Delta^{\mathcal{J}}$, and $A \in \mathbf{C}$;
- whenever $\delta \;\sigma\; \epsilon$ and $(\delta, \zeta) \in r^{\mathcal{I}}$, then there exists an $\eta \in \Delta^{\mathcal{J}}$ such that $\zeta \;\sigma\; \eta$ and $(\epsilon, \eta) \in r^{\mathcal{J}}$, for all $\delta, \zeta \in \Delta^{\mathcal{I}}$, $\epsilon \in \Delta^{\mathcal{J}}$, and $r \in \mathbf{R}$.

We write $(\mathcal{I}, \delta) \precsim (\mathcal{J}, \epsilon)$ if there is a simulation σ from $\mathcal{I}$ to $\mathcal{J}$ such that $\delta \;\sigma\; \epsilon$.

(a) Show that $(\mathcal{I}, \delta) \sim (\mathcal{J}, \epsilon)$ implies $(\mathcal{I}, \delta) \precsim (\mathcal{J}, \epsilon)$ and $(\mathcal{J}, \epsilon) \precsim (\mathcal{I}, \delta)$.

(b) Is the converse of the implication in (a) also true?

(c) Show that, if $(\mathcal{I}, \delta) \precsim (\mathcal{J}, \epsilon)$, then $\delta \in C^{\mathcal{I}}$ implies $\epsilon \in C^{\mathcal{J}}$ for all $\mathcal{EL}$ concept descriptions C.

(d) Which of the constructors disjunction, negation, or value restriction can be added to $\mathcal{EL}$ without losing the property in (c)?

(e) Show that $\mathcal{ALC}$ is more expressive than $\mathcal{EL}$.

(f) Show that $\mathcal{ELI}$ is more expressive than $\mathcal{EL}$.

(g) Can the fact that subsumption in $\mathcal{EL}$ is decidable in polynomial time, while subsumption in $\mathcal{ELI}$ is ExpTime-complete, be used to show that $\mathcal{ELI}$ is more expressive than $\mathcal{EL}$?

Exercise 11.4 We consider the description logic $\mathcal{EL}_{\mathsf{si}}$ that extends $\mathcal{EL}$ by concept descriptions of the form $\exists^{\mathsf{sim}}(\mathcal{I},\delta)$ where $\mathcal{I}$ is a finite interpretation and $\delta \in \Delta^{\mathcal{I}}$. Their semantics is defined as follows.

$$(\exists^{\mathsf{sim}}(\mathcal{I},\delta))^{\mathcal{J}} \coloneqq \{\, \epsilon \mid \epsilon \in \Delta^{\mathcal{J}} \text{ and } (\mathcal{I},\delta) \precsim (\mathcal{J},\epsilon) \,\}$$

Concept inclusions are then defined as usual.

(a) Show that each $\mathcal{EL}_{\mathsf{si}}$ concept description is equivalent to some concept description of the form $\exists^{\mathsf{sim}}(\mathcal{I},\delta)$.

(b) Show that $\mathcal{EL}_{\mathsf{si}}$ is more expressive than $\mathcal{EL}$.

(c) Show that checking subsumption in $\mathcal{EL}_{\mathsf{si}}$ without any TBox can be done in polynomial time.