Exercise 12.1  Let $\mathcal{T}_1$ be an $\mathcal{EL}$ TBox, $C, D \in \mathcal{EL}$ concept descriptions, and $\mathcal{T}_2 := \mathcal{T}_1 \cup \{ A \sqsubseteq C, D \sqsubseteq B \}$, where $A, B$ are new concept names (as in Lemma 6.1). Show that $\mathcal{T}_2$ is a conservative extension of $\mathcal{T}_1$. Is this still the case after adding $A \sqsubseteq B$ to $\mathcal{T}_2$? What about adding $B \sqsubseteq A$?

Exercise 12.2  Consider the $\mathcal{EL}$ TBox

\[
\mathcal{T} := \left\{ \begin{array}{l}
A \sqsubseteq B \cap \exists r.C, \\
B \cap \exists r.B \sqsubseteq C \cap D, \\
C \sqsubseteq (\exists r.A) \cap B, \\
(\exists r.\exists r.B) \cap D \sqsubseteq \exists r.(A \cap B)
\end{array} \right\},
\]

where $A, B, C, D$ are concept names. Use the classification procedure for $\mathcal{EL}$ to check whether the following subsumption relationships hold true w.r.t. $\mathcal{T}$.

(a) $A \sqsubseteq B$

(b) $A \sqsubseteq \exists r.\exists r.A$

(c) $B \cap \exists r.A \sqsubseteq \exists r.C$

Exercise 12.3  Consider the $\mathcal{ELI}$ TBox

\[
\mathcal{T} := \left\{ \begin{array}{l}
A_1 \cap A_2 \sqsubseteq \exists r.B, \\
\exists r.A_2 \sqsubseteq C, \\
A \sqsubseteq A_1 \cap A_2, \\
\exists r.(B \cap C) \sqsubseteq D
\end{array} \right\},
\]

where $A, A_1, A_2, B, C, D$ are concept names. Use the classification procedure for $\mathcal{ELI}$ to check whether the following subsumption relationships hold true w.r.t. $\mathcal{T}$.

(a) $A \sqsubseteq D$

(b) $\exists r.A \sqsubseteq \exists r.D$

(c) $A \sqsubseteq \exists r.A$