TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Automata and Logic

Winter Semester 2018 / 2019

Exercise Sheet 1

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Regular Languages, Finite Monoids, and Logical Formulae

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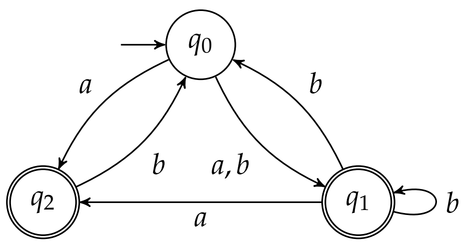
Exercise 1.1 Fix the alphabet $\Sigma := \{a, b\}$ and let $\alpha := a^+b^* + b^+a^*$ be a regular expression over Σ. Give a regular expression β for the complement language of α, i.e., find some β describing the set of words over Σ that are not expressed by α.

Exercise 1.2 Consider the non-deterministic finite automaton $\mathcal{A} := (\{q_0, q_1, q_2\}, \{a, b\}, \{q_0\}, \Delta, \{q_1, q_2\})$ the transition relation Δ of which is graphically described below.



Apply the power-set construction to $\mathcal{A}$ in order to obtain a *deterministic* finite automaton that accepts the same language as $\mathcal{A}$.

Exercise 1.3 For a language $L \subseteq \Sigma^*$ over some finite alphabet Σ, the *Nerode right congruence* ρ_L is defined as follows. For any $u, v \in \Sigma^*$, it holds true that

$$u \; \rho_L \; v \text{ if, and only if, } uw \in L \Leftrightarrow vw \in L \text{ for all } w \in \Sigma^*.$$

We define the deterministic finite automaton $\mathcal{A}_L := (Q_L, \Sigma, q_L, \delta_L, F_L)$ with the following components.

$Q_L := \{\, [u]_{\rho_L} \mid u \in \Sigma^* \,\}$ where $[u]_{\rho_L} := \{\, v \in \Sigma^* \mid u \; \rho_L \; v \,\}$

$q_L := [\varepsilon]_{\rho_L}$ where ε denotes the empty word

$\delta_L([u]_{\rho_L}, a) := [ua]_{\rho_L}$ for $u \in \Sigma^*$ and $a \in \Sigma$

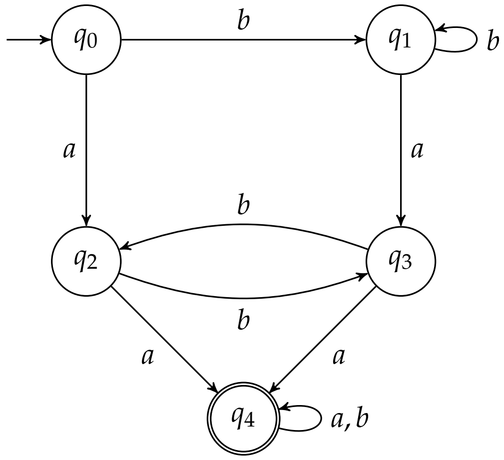
$F_L := \{\, [u]_{\rho_L} \mid u \in L \,\}$

Show that, for each regular language L, the following statements hold true.

(a) $\mathcal{A}_L$ is well-defined.

(b) $\mathcal{A}_L$ is minimal (w.r.t. the number of states), i.e., for every deterministic automaton $\mathcal{A} = (Q, \Sigma, q_0, \delta, F)$ with $L(\mathcal{A}) = L$, we have $|Q_L| \leq |Q|$.

Exercise 1.4 Let $\mathcal{A}$ be the finite automaton that accepts words over the alphabet $\Sigma := \{a, b\}$ and is graphically described as follows.



Construct a minimal automaton $\mathcal{A}'$ such that $L(\mathcal{A}') = L(\mathcal{A})$.

Exercise 1.5 Prove the following claims by devising appropriate decision procedures.

(a) The emptiness problem for regular languages is decidable.

(b) The inclusion problem for regular languages is decidable.