TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Automata and Logic

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Exercise Sheet 4

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Regular Languages, Finite Monoids, and Logical Formulae

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Exercise 4.1 Show that any submonoid of a finite group is also a group.

Exercise 4.2 Let V be an M-variety. Show that $L(V)_\Sigma$ is closed under union *without* using Proposition 1.22.

Exercise 4.3 Let Σ be a finite alphabet. Prove or refute the following claims.

(a) Every regular language $L \subseteq \Sigma^*$ is accepted by its syntactic monoid.

(b) If $L \subseteq \Sigma^*$ is accepted by a finite group, then the syntactic monoid of L is a finite group.

(c) For every regular language $L \subseteq \Sigma^*$, the syntactic monoid M_L is the smallest monoid accepting L, i.e., for every monoid M accepting L, we have $|M_L| \leq |M|$.

(d) For a word $w = a_1 \dots a_n$, let $\overleftarrow{w}$ denote the mirror image of w, i.e., $\overleftarrow{w} := a_n \dots a_1$. For a language $L \subseteq \Sigma^*$, we define $\overleftarrow{L} := \{\overleftarrow{w} \mid w \in L\}$. **Claim:** If the minimal automaton for L has n states, then the minimal automaton for $\overleftarrow{L}$ has also n states.

Exercise 4.4 Let L_1 be the language over $\{a\}$ described by a^+, and let L_2 be the language over $\{a, b\}$ described by $(a + b)^* b (a + b)^*$.

(a) Is there a monoid that accepts both L_1 and L_2?

(b) Are the syntactic monoids of those languages isomorphic?

Exercise 4.5 Let L_1 and L_2 be two languages over the same finite alphabet Σ that are accepted by the same monoid $(M, \circ, 1)$. Prove or refute the following statements.

(a) M accepts $L_1 \cap L_2$.

(b) M accepts $L_1 \cup L_2$.

(c) M accepts $L_1 \cdot L_2$.

Exercise 4.6 Let V be the M-variety of all commutative finite groups. Show that there exists a language $L \subseteq \{a\}^*$ such that $L \in L(V)_{\{a\}}$ but $L \notin L(V)_{\{a,b\}}$.