Automata and Logic

Winter Semester 2018 / 2019

Exercise Sheet 5 14th November 2018

Regular Languages, Finite Monoids, and Logical Formulae

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Exercise 5.1 Prove or refute the following claim. There is a language $L \subseteq \{a, b\}^*$ such that its syntactic semigroup S_L and its syntactic monoid M_L are isomorphic.

Exercise 5.2 (a) For each of the following words over the alphabet $\{0, 1\}^k$, give a corresponding interpretation over the predicate symbols $P_1, \ldots, P_k$ as discussed in the lecture.

$$\begin{aligned} k = 2&: \quad (1,1), (1,1), (0,1), (1,0) \\ k = 3&: \quad (0,0,0), (1,0,0), (0,1,0), (0,0,1) \\ k = 3&: \quad (1,1,0), (1,0,1), (1,1,1), (1,1,0) \end{aligned}$$

(b) Describe all interpretations that correspond to words of the language

$$L(((0,1) \cdot (1,0))^+) \subseteq (\{0,1\}^2)^+.$$

Exercise 5.3 Let $\Sigma := \{a, b\}$. For each of the following regular expressions r_i, give a first-order formula ϕ_i such that $L(r_i) \setminus \{\varepsilon\} = L(\phi_i)$.

(a) $r_1 := \Sigma^*$

(b) $r_2 := \varepsilon$

(c) $r_3 := (abb^*)^*$

(d) $r_4 := a^*b^* + b^*a^*$

(e) $r_5 := (aaa \cdot \Sigma^*) + b^*$

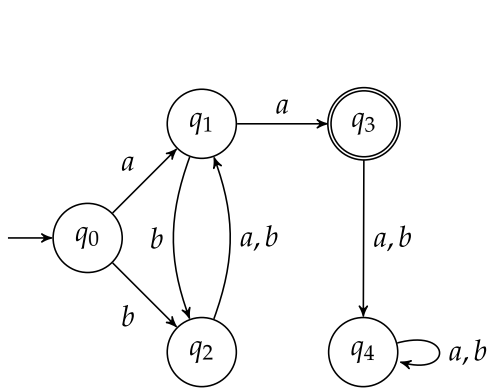
Exercise 5.4 Let $(S, \circ)$ be a finite semigroup, $m \in S$, and $i, k, \ell \in \mathbb{N} \setminus \{0\}$ defined as in the proof of Proposition 2.4. Show that if k is minimal with respect to the property described in the proof, then

$$(\{m^i, \ldots, m^{i+k-1}\}, \circ, m^\ell)$$

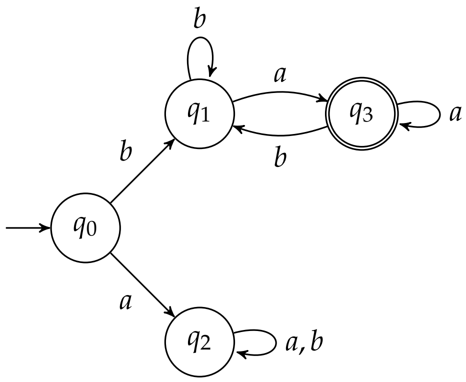
is a group. Is it still a group if k is not minimal?

Exercise 5.5 Let $\Sigma := \{a, b\}$ and L_1, L_2 be the languages accepted by the automata displayed below.

(1)



(2)



Use the proof of Corollary 2.10 to show that $L_1 \notin (B_0)_\Sigma$ and $L_2 \in (B_0)_\Sigma$. Moreover, represent L_2 as a Boolean combination of languages from the set

$$\{u\Sigma^* \mid u \in \Sigma^*\} \cup \{\Sigma^* u \mid u \in \Sigma^*\}.$$

Exercise 5.6 Prove or refute the following claims.

(a) For every finite alphabet Σ and any word $w \in \Sigma^*$, we have $\{w\} \in (B_0)_\Sigma$.

(b) For every two finite alphabets Σ and Σ' with $\Sigma \subseteq \Sigma'$, and every language $L \subseteq \Sigma^*$, we have: if $L \in (B_0)_\Sigma$, then $L \in (B_0)_{\Sigma'}$.

(c) Let $(M, \circ, 1)$ be a monoid, where 1 is the only idempotent element of M. Then $(M, \circ, 1)$ is a group.

(d) Let $(S, \circ)$ be a semigroup with $e \in S$ being idempotent. Then $(eSe, \circ, e)$ is the largest submonoid of S with e as unit element.

(e) Let $(S, \circ) \in \widehat{\mathbb{D}}$. If there exists an element $s \in S$ such that $(S, \circ, s)$ is a monoid, then $|S| = 1$.