TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Automata and Logic

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Exercise Sheet 7

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Star-Free Languages

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Exercise 7.1 Let $\Sigma := \{a\}$. Recall from Definition 3.10 that $L_{k,n}$ denotes the set of all first-order formulae over the signature $\{=, <, Q_a\}$ containing k free variables and having quantifier depth at most n. For the following combinations of k and n, determine a *finite* set $\Gamma_{k,n}$ such that, for every formula $\phi \in L_{k,n}$, there is a formula $\psi \in \Gamma_{k,n}$ with $\phi \equiv \psi$. Determine also the equivalence classes of $\equiv_{k,n}$.

(a) $k = 1, n = 0$

(b) $k = 2, n = 0$

(c) $k = 0, n = 1$

(d) $k = 1, n = 1$

For each of the following formulae, find an equivalent finite disjunction of suitable formulae ϕ_W where W is some equivalence class of $\equiv_{2,0}$.

(i) true

(ii) $\neg(x < y) \vee x = y$

(iii) false

Exercise 7.2 Consider the Ehrenfeucht-Fraïssé games on the following words.

(a) ab and ba

(b) $aaabaaa$ and $aabaaa$

For each case, determine the smallest number k such that Player I has a winning strategy in k moves.

Exercise 7.3 Consider the Ehrenfeucht-Fraïssé games on the words a^i and a^j with $i < j$.

(a) Describe an optimal winning strategy for Player I, i.e., a strategy such that Player I wins with a minimal number of moves.

(b) Prove that Player I has a winning strategy on a^i and a^j in m moves if $i < 2^m - 1$.