Automata and Logic

Winter Semester 2018 / 2019

Exercise Sheet 8 5th December 2018

Infinite Words and Büchi-Automata

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Exercise 8.1 Fix the alphabet $\Sigma := \{a, b\}$, and let the language $L \subseteq \Sigma^*$ be defined by the regular expression $(a^*bb^*)^*$. Show that the following equation holds true.

$$\lim L = \{\, \alpha \in \Sigma^\omega \mid \text{if } \alpha(i) = a \text{, then there is some } j > i \text{ with } \alpha(j) = b \,\}$$

Exercise 8.2 For each of the following ω-regular languages over the alphabet $\Sigma := \{a, b, c\}$, check whether it is ω-regular and if so, devise a Büchi-automaton that recognizes it.

(a) $L_1 := \{\, \alpha \in \Sigma^\omega \mid \exists i \in \mathbb{N} \colon \alpha(i, i+2) = abc \,\}$

(b) $L_2 := \{\, \alpha \in \Sigma^\omega \mid \{\, i \in \mathbb{N} \mid \alpha(i, i+2) = abc \,\} \text{ is infinite} \,\}$

(c) $L_3 := (a^+b^+c^+)^\omega$

Exercise 8.3 Let Σ be a finite alphabet, and consider languages $L, L_1, L_2 \subseteq \Sigma^*$. Prove or refute each of the following claims.

(a) $(L_1 \cup L_2)^\omega \subseteq L_1^\omega \cup L_2^\omega$

(b) $(L_1 \cup L_2)^\omega \supseteq L_1^\omega \cup L_2^\omega$

(c) $\lim(L_1 \cup L_2) \subseteq \lim L_1 \cup \lim L_2$

(d) $\lim(L_1 \cup L_2) \supseteq \lim L_1 \cup \lim L_2$

(e) $L^\omega \subseteq \lim L^+$

(f) $L^\omega \supseteq \lim L^+$

(g) $\lim(L_1 \cdot L_2) \subseteq L_1 \cdot L_2^\omega$

(h) $\lim(L_1 \cdot L_2) \supseteq L_1 \cdot L_2^\omega$

Exercise 8.4 Prove the following statements.

(a) If $L \subseteq \Sigma^+$ is regular for some finite alphabet Σ, then there exists a non-deterministic finite automaton $\mathcal{A}$ with only *one* final state such that $L = L(\mathcal{A})$.

(b) If $L \subseteq \Sigma^*$ is regular for some finite alphabet Σ, then there exists a non-deterministic finite automaton $\mathcal{A}$ with at most *two* final states such that $L = L(\mathcal{A})$.

(c) There is *no* $k \geq 1$ such that, if $L \subseteq \Sigma^\omega$ is ω-regular for some finite alphabet Σ, then there exists a Büchi-automaton $\mathcal{A}$ with at most k final states such that $L = L_\omega(\mathcal{A})$.

Hint. Consider the languages $a^\omega \cup b^\omega, a^\omega \cup b^\omega \cup c^\omega, \ldots$.

Exercise 8.5 For each finite non-deterministic automaton $\mathcal{A}$, let $\mathcal{A}_{\text{det}}$ denote the minimal deterministic finite automaton such that $L(\mathcal{A}) = L(\mathcal{A}_{\text{det}})$. Prove or refute the following claims.

(a) $\lim L(\mathcal{A}) \subseteq L_\omega(\mathcal{A}_{\text{det}})$

(b) $\lim L(\mathcal{A}) \supseteq L_\omega(\mathcal{A}_{\text{det}})$

(c) $L_\omega(\mathcal{A}) \subseteq L_\omega(\mathcal{A}_{\text{det}})$

(d) $L_\omega(\mathcal{A}) \supseteq L_\omega(\mathcal{A}_{\text{det}})$