Exercise 9.1  (a) Show that the construction used in the proof of Statement 1 in Lemma 4.7 does not work for automata in which the initial state is reachable from another state.

(b) Complete the proof of Lemma 4.7 by showing that \( L_1 \cup L_2 \) is Büchi-recognizable if \( L_1, L_2 \subseteq \Sigma^\omega \) are Büchi-recognizable.

Exercise 9.2  Consider the alphabet \( \Sigma := \{a, b\} \), and let \( L \subseteq \Sigma^\omega \) be the \( \omega \)-language recognized by the following Büchi-automaton.

![Büchi-Automaton Diagram]

Find a number \( n \geq 1 \) and regular languages \( U_1, V_1, \ldots, U_n, V_n \subseteq \Sigma^* \) such that

\[
\bigcup_{i=1}^{n} U_i \cdot V_i^\omega = L.
\]
Exercise 9.3  Consider Büchi-automata with the following transition relation.

Check whether the recognized $\omega$-language is empty for the following sets of final states.

(a) $\{q_0, q_1\}$
(b) $\{q_2, q_3\}$
(c) $\{q_1, q_3\}$

Exercise 9.4  Fix some finite alphabet $\Sigma$. Prove that, for every $\omega$-regular language $L$ over $\Sigma$, there is a Büchi-automaton $\mathcal{A}$ such that $L_{\omega}(\mathcal{A}) = L$ holds true and, for each state $q$ of $\mathcal{A}$ and for each symbol $a \in \Sigma$, there are at most two transitions of $\mathcal{A}$ that start in $q$ and read $a$. 