



Automata and Logic

Winter Semester 2018 / 2019

Exercise Sheet 9

12th December 2018

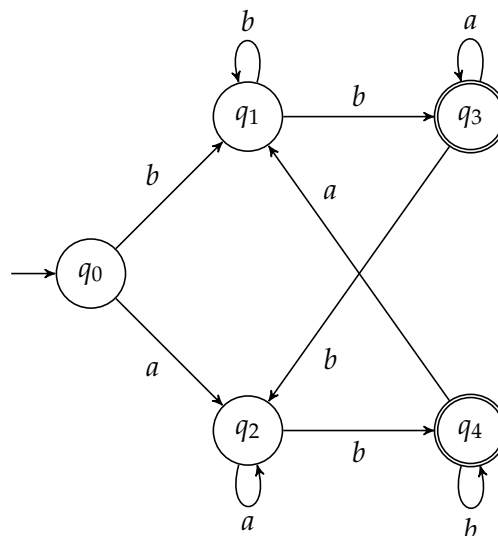
Infinite Words and Büchi-Automata

PD Dr.-Ing. habil. Anni-Yasmin Turhan, Dipl.-Math. Francesco Kriegel

Exercise 9.1 (a) Show that the construction used in the proof of Statement 1 in Lemma 4.7 does not work for automata in which the initial state is reachable from another state.

(b) Complete the proof of Lemma 4.7 by showing that $L_1 \cup L_2$ is Büchi-recognizable if $L_1, L_2 \subseteq \Sigma^\omega$ are Büchi-recognizable.

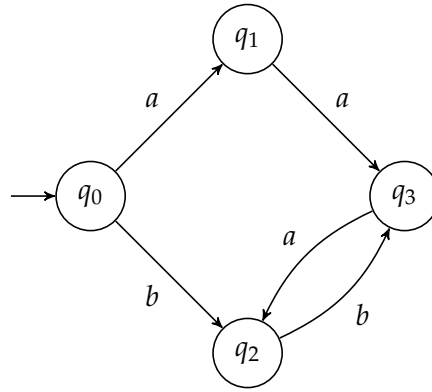
Exercise 9.2 Consider the alphabet $\Sigma := \{a, b\}$, and let $L \subseteq \Sigma^\omega$ be the ω -language recognized by the following Büchi-automaton.



Find a number $n \geq 1$ and regular languages $U_1, V_1, \dots, U_n, V_n \subseteq \Sigma^*$ such that

$$\bigcup_{i=1}^n U_i \cdot V_i^\omega = L.$$

Exercise 9.3 Consider Büchi-automata with the following transition relation.



Check whether the recognized ω -language is empty for the following sets of final states.

- (a) $\{q_0, q_1\}$
- (b) $\{q_2, q_3\}$
- (c) $\{q_1, q_3\}$

Exercise 9.4 Fix some finite alphabet Σ . Prove that, for every ω -regular language L over Σ , there is a Büchi-automaton \mathcal{A} such that $L_\omega(\mathcal{A}) = L$ holds true and, for each state q of \mathcal{A} and for each symbol $a \in \Sigma$, there are *at most two* transitions of \mathcal{A} that start in q and read a .