TECHNISCHE
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Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

## Automata and Logic

Winter Semester 2018/2019

## Exercise Sheet 10

Infinite Words and Büchi-Automata
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Exercise 10.1 Show that, for each sequence $\left(r_{n}\right)_{n \geq 0}$ of real numbers, there exists some infinite subsequence that is either strictly increasing, strictly decreasing, or constant.

Exercise 10.2 Define $\Sigma:=\{a, b\}$, and let $L \subseteq \Sigma^{\omega}$ be the $\omega$-language recognized by the following Büchi-automaton.


Use the method from Corollary 4.22 to construct a Büchi-automaton that recognizes the complement language $\Sigma^{\omega} \backslash L$.

Exercise 10.3 Let $\Sigma:=\{a, b, c\}$ and consider the following transition system.


We derive four Muller automata $\mathcal{A}_{1}, \mathcal{A}_{2}, \mathcal{A}_{3}$, and $\mathcal{A}_{4}$ by selecting corresponding sets of final states $\mathcal{F}_{1}$, $\mathcal{F}_{2}, \mathcal{F}_{3}$, and $\mathcal{F}_{4}$, respectively, as follows.
(a) $\mathcal{F}_{1}:=\left\{\left\{q_{0}, q_{3}\right\},\left\{q_{3}\right\}\right\}$
(b) $\mathcal{F}_{2}:=\left\{\left\{q_{0}, q_{1}\right\},\left\{q_{2}\right\}\right\}$
(c) $\mathcal{F}_{3}:=\left\{\left\{q_{0}, q_{1}, q_{2}\right\}\right\}$
(d) $\mathcal{F}_{4}:=\left\{\left\{q_{0}\right\},\left\{q_{0}, q_{1}\right\},\left\{q_{2}\right\},\left\{q_{0}, q_{1}, q_{2}\right\}\right\}$

For each index $i \in\{1, \ldots, 4\}$, determine the $\omega$-language $L_{\omega}\left(\mathcal{A}_{i}\right)$.

