

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Automata and Logic

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Exercise Sheet 10 Infinite Words and Büchi-Automata

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Exercise 10.1 Show that, for each sequence $(r_n)_{n\geq 0}$ of real numbers, there exists some infinite subsequence that is either strictly increasing, strictly decreasing, or constant.

Hiut If you do not want to show the claim directly, you could use Ramsey's Theorem.

Exercise 10.2 Define $\Sigma := \{a, b\}$, and let $L \subseteq \Sigma^{\omega}$ be the ω -language recognized by the following Büchi-automaton.



Use the method from Corollary 4.22 to construct a Büchi-automaton that recognizes the complement language $\Sigma^{\omega} \setminus L$.

Exercise 10.3 Let $\Sigma := \{a, b, c\}$ and consider the following transition system.



We derive four Muller automata A_1 , A_2 , A_3 , and A_4 by selecting corresponding sets of final states \mathcal{F}_1 , \mathcal{F}_2 , \mathcal{F}_3 , and \mathcal{F}_4 , respectively, as follows.

- (a) $\mathcal{F}_1 \coloneqq \{\{q_0, q_3\}, \{q_3\}\}$
- (b) $\mathcal{F}_2 \coloneqq \{\{q_0, q_1\}, \{q_2\}\}$
- (c) $\mathcal{F}_3 \coloneqq \{\{q_0, q_1, q_2\}\}$
- (d) $\mathcal{F}_4 \coloneqq \{\{q_0\}, \{q_0, q_1\}, \{q_2\}, \{q_0, q_1, q_2\}\}$

For each index $i \in \{1, ..., 4\}$, determine the ω -language $L_{\omega}(\mathcal{A}_i)$.