TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Automata and Logic

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Exercise Sheet 11
Infinite Words and Logical Formulae

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Exercise 11.1 For each of the following languages L_i, give an S1S-formula ϕ_i such that $L_\omega(\phi_i) = L_i$ holds true.

(a) $L_1 \coloneqq (abb^*)^\omega$

(b) $L_2 \coloneqq ((aa)^+(bb)^+)^\omega$

(c) $L_3 \coloneqq (aaa)^+b(a \cup b)^\omega$

Exercise 11.2 Transform the S1S-formula $P(\underline{0})$ into an equivalent S1S$_0$-formula.

Exercise 11.3 Fix the ω-language $L \coloneqq (a^+b)^\omega \cup (b^+a)^\omega$. Use the proof of Proposition 5.4 to construct a closed S1S-formula ϕ that satisfies $L_\omega(\phi) = L$.

Exercise 11.4 A *Rabin-automaton* is a tuple $\mathcal{A} \coloneqq (Q, \Sigma, I, \Delta, \Omega)$ where Q, Σ, I, and Δ are defined as for non-deterministic Büchi-automata, and

$$\Omega \coloneqq \{(F_1, G_1), \ldots, (F_n, G_n)\}$$

is a finite set of pairs (F_i, G_i) such that $F_i, G_i \subseteq Q$.

For a word α, let $\mathsf{path}_{\mathcal{A}}(\alpha)$ denote the set of all paths in $\mathcal{A}$ labeled with α. For a path $p \in \mathsf{path}_{\mathcal{A}}(\alpha)$, let $\mathsf{inf}(p)$ denote the set of all states that are visited infinitely often. The ω-language $L_\omega(\mathcal{A})$ recognized by $\mathcal{A}$ is defined as

$$L_\omega(\mathcal{A}) \coloneqq \{\, \alpha \in \Sigma^\omega \mid \exists i \in \{1, \ldots, n\}\ \exists p \in \mathsf{path}_{\mathcal{A}}(\alpha)\colon \mathsf{inf}(p) \cap F_i \neq \emptyset \text{ and } \mathsf{inf}(p) \cap G_i = \emptyset \,\}.$$

Show that every language recognized by a Rabin-automaton is also recognized by a Büchi-automaton.

Hint. Fix some arbitrary Rabin-automaton $\mathcal{A}$ and construct an S1S-formula $\phi_{\mathcal{A}}$ defining the language $L_\omega(\mathcal{A})$.