TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Automata and Logic

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Exercise Sheet 13

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Automata on Infinite Trees
Tree-Automata and Logical Formulae

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Exercise 13.1 Let $\Sigma := \{a, b\}$ be an alphabet with two binary symbols and consider the language

$$L := \{ t \in \mathbf{T}_\Sigma^\omega \mid \text{there is a path in } t \text{ containing the symbol } a \text{ only finitely often} \}.$$

(a) Is L Rabin-recognizable?

(b) Is L Büchi-recognizable?

Exercise 13.2 Let Σ be a finite alphabet with arity function containing at least two binary symbols f and g. Is the ω-tree language $L := \{ f(t, t) \mid t \in \mathbf{T}_\Sigma^\omega \}$ Büchi-recognizable? Justify your answer.

Exercise 13.3 Let Σ be a finite alphabet of binary symbols. For each ω-tree $t \in \mathbf{T}_\Sigma^\omega$, we define an ω-language $\mathsf{path}(t) \subseteq \Sigma^\omega$ as follows:

$$\mathsf{path}(t) := \{ \alpha \in \Sigma^\omega \mid \text{there is a path } i_0 i_1 i_2 \ldots \text{ in } t \text{ such that } t(i_0 i_1 \ldots i_n) = \alpha(n) \text{ for all } n \in \mathbb{N} \}.$$

Furthermore, let

$$\mathsf{path}(B) := \bigcup \{ \mathsf{path}(t) \mid t \in B \}$$

for each $B \subseteq \mathbf{T}_\Sigma^\omega$. For an ω-language $L \subseteq \Sigma^\omega$, let

$$\mathsf{tree}(L) := \{ t \in \mathbf{T}_\Sigma^\omega \mid \mathsf{path}(t) \subseteq L \}.$$

Prove or refute each of the following claims.

(a) $\mathsf{tree}(\mathsf{path}(B)) = B$

(b) $\mathsf{path}(\mathsf{tree}(L)) = L$

(c) If B is Büchi-recognizable, then $\mathsf{path}(B)$ is also Büchi-recognizable.

(d) If $\mathsf{path}(B)$ is Büchi-recognizable, then B is also Büchi-recognizable.

(e) If L is Büchi-recognizable, then $\mathsf{tree}(L)$ is also Büchi-recognizable.

(f) If $\mathsf{tree}(L)$ is Büchi-recognizable, then L is also Büchi-recognizable.

Exercise 13.4 Let $\Sigma := \{0, 1\}$ and define

$$L := \{\, t \in \mathbf{T}_\Sigma^\omega \mid \text{for every path } p \text{, if } p \text{ contains the symbol } 0, \\ \text{then } p \text{ contains the symbol } 1 \text{ only finitely often} \,\}.$$

Give an S2S-formula ϕ such that $L_\omega(\phi) = L$.

Exercise 13.5 Let $\Sigma := \{a, b\}$ and consider the Rabin-automaton

$$\mathcal{A} := (\{q_0, q_1, q_2\}, \Sigma, \{q_2\}, \Delta, \{(\{q_0, q_2\}, \{q_1\})\})$$

for which the transition assignment Δ is defined in the following way.

$$\begin{aligned} \Delta_a \colon q_0 &\mapsto \{(q_1, q_1)\} & \qquad \Delta_b \colon q_0 &\mapsto \{(q_0, q_0)\} \\ q_1 &\mapsto \{(q_1, q_1)\} & q_1 &\mapsto \{(q_0, q_0)\} \\ q_2 &\mapsto \{(q_0, q_1)\} & q_2 &\mapsto \{(q_0, q_1)\} \end{aligned}$$

Find an S2S-formula ϕ satisfying $L_\omega(\mathcal{A}) = L_\omega(\phi)$.

Exercise 13.6 Consider the ω-tree language L over a finite alphabet Σ that is defined as

$$L := \{\, t \in \mathbf{T}_\Sigma^\omega \mid t \text{ is a strict subtree of } t \,\}.$$

For each of the following claims, decide whether it holds true and justify your answer.

(a) There is a tree $t \in L$ such that it contains itself only finitely often as a strict subtree.

(b) L is Büchi-recognizable.

(c) L is Rabin-recognizable.

(d) There is a closed S2S-formula ϕ such that $L_\omega(\phi) = L$ is satisfied.