Problem 1: View-based Identity Problem

Obtains Views \[\text{asks queries w.r.t. } \Omega\]

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Talk in Bolzano 2018 2 / 19
Problem 1: View-based Identity Problem

- A view $V$ is a finite collection of queries together with their answers
- Consider subsumption & conjunctive queries (SELECT-JOIN-PROJECT in DBs)
Problem 1: View-based Identity Problem

- A view $V$ is a finite collection of queries together with their answers.
- Consider subsumption & conjunctive queries (\texttt{SELECT-JOIN-PROJECT} in DBs).

Given an ontology $\mathcal{O}_I$

- At rôle $\hat{r}_1$
  - queries through $\mathcal{O}_{\hat{r}_1} \subseteq \mathcal{O}_I$ \hspace{1cm} \textbf{switch} \hspace{1cm} \ldots \hspace{1cm} \textbf{switch}
  - obtains View $V_{\hat{r}_1}$

- At rôle $\hat{r}_k$
  - queries through $\mathcal{O}_{\hat{r}_k} \subseteq \mathcal{O}_I$
  - obtains View $V_{\hat{r}_k}$

Is the identity of an anonymous $x$ hidden w.r.t. $V_{\hat{r}_1}, \ldots, V_{\hat{r}_k}$?
Ontologies

Given an ontology $\mathcal{O}_I$

At rôle $\hat{r}_1$
- queries through $\mathcal{O}_{\hat{r}_1} \subseteq \mathcal{O}_I$ \quad switch \ldots \quad switch
- obtains View $V_{\hat{r}_1}$

At rôle $\hat{r}_k$
- queries through $\mathcal{O}_{\hat{r}_k} \subseteq \mathcal{O}_I$
- obtains View $V_{\hat{r}_k}$

Is the identity of an anonymous $x$ hidden w.r.t. $V_{\hat{r}_1}, \ldots, V_{\hat{r}_k}$?

An ontology $\mathcal{O}$ consists of TBox $\mathcal{T}$ and ABox $\mathcal{A}$.

- A TBox $\mathcal{T}$ is a set of General Concept Inclusions (GCIs) $C \sqsubseteq D$
  → background knowledge
Given an **ontology** $\mathcal{O}$.

At rôle $\hat{r}_1$:
- queries through $\mathcal{O}_{\hat{r}_1} \subseteq \mathcal{O}$
- obtains View $V_{\hat{r}_1}$

At rôle $\hat{r}_k$:
- queries through $\mathcal{O}_{\hat{r}_k} \subseteq \mathcal{O}$
- obtains View $V_{\hat{r}_k}$

Is the identity of an anonymous $x$ hidden w.r.t. $V_{\hat{r}_1}, \ldots, V_{\hat{r}_k}$?

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An **ontology** $\mathcal{O}$ consists of **TBox** $\mathcal{T}$ and **ABox** $\mathcal{A}$.

- A TBox $\mathcal{T}$ is a set of **General Concept Inclusions (GCIs)** $C \sqsubseteq D$ → background knowledge
- An ABox $\mathcal{A}$ is a set of **concept assertions** $C(a)$ and **relationship assertions** $r(a, b)$ → knowledge about individuals
Given an ontology $\mathcal{O}_I$

At rôle $\hat{r}_1$
- queries through $\mathcal{O}_{\hat{r}_1} \subseteq \mathcal{O}_I$ \[\xrightarrow{\text{switch}}\] \[\xrightarrow{\text{switch}}\] \[\xrightarrow{\text{switch}}\]
- obtains View $V_{\hat{r}_1}$

At rôle $\hat{r}_k$
- queries through $\mathcal{O}_{\hat{r}_k} \subseteq \mathcal{O}_I$
- obtains View $V_{\hat{r}_k}$

Is the identity of an anonymous $\times$ hidden w.r.t. $V_{\hat{r}_1}, \ldots, V_{\hat{r}_k}$?
Queries

Given an ontology $\mathcal{O}_I$

At rôle $\hat{r}_1$
- **queries** through $\mathcal{O}_{\hat{r}_1} \subseteq \mathcal{O}_I$ \(\xrightarrow{\text{switch}}\) $\ldots$ \(\xrightarrow{\text{switch}}\)
- obtains View $V_{\hat{r}_1}$

At rôle $\hat{r}_k$
- **queries** through $\mathcal{O}_{\hat{r}_k} \subseteq \mathcal{O}_I$
- obtains View $V_{\hat{r}_k}$

Is the identity of an anonymous $x$ hidden w.r.t. $V_{\hat{r}_1}, \ldots, V_{\hat{r}_k}$?

1. **Subsumption Query**: $q = C \sqsubseteq D$, where $C$ and $D$ are DL concepts

2. **Conjunctive Query**: $q(x) \leftarrow \exists \bar{y}. \text{conj}(\bar{x}, \bar{y})$, where
   - $\bar{x}$ are **answer variables** and $\bar{y}$ are **existentially quantified variables**.
   - $\text{conj}(\bar{x}, \bar{y})$ is a **conjunction of atoms** $A(z)$ or $r(z, z')$. 

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Given an ontology $\mathcal{O}_I$

At rôles $\hat{r}_1$
- queries through $\mathcal{O}_{\hat{r}_1} \subseteq \mathcal{O}_I$ \[\text{switch} \quad \ldots \quad \text{switch}\]
- obtains **View** $V_{\hat{r}_1}$

At rôles $\hat{r}_k$
- queries through $\mathcal{O}_{\hat{r}_k} \subseteq \mathcal{O}_I$
- obtains **View** $V_{\hat{r}_k}$

Is the identity of an anonymous $x$ hidden w.r.t. $V_{\hat{r}_1}, \ldots, V_{\hat{r}_k}$?
Given an ontology $\mathcal{O}_I$

At rôle $\hat{r}_1$
- queries through $\mathcal{O}_{\hat{r}_1} \subseteq \mathcal{O}_I$ switch $\ldots$ switch
- obtains View $V_{\hat{r}_1}$

At rôle $\hat{r}_k$
- queries through $\mathcal{O}_{\hat{r}_k} \subseteq \mathcal{O}_I$
- obtains View $V_{\hat{r}_k}$

Is the identity of an anonymous $x$ hidden w.r.t. $V_{\hat{r}_1}, \ldots, V_{\hat{r}_k}$?

Let $q = C \sqsubseteq D$ be a subsumption query. The answer for $q$ w.r.t. a rôle $\hat{r}$ is $\{true\}$ if

$$C^\mathcal{I} \subseteq D^\mathcal{I} \text{ for all models } \mathcal{I} \text{ of } \mathcal{O}_{\hat{r}}.$$
Given an ontology $\mathcal{O}_I$

At rôle $\hat{r}_1$
- queries through $\mathcal{O}_{\hat{r}_1} \subseteq \mathcal{O}_I$ \[\xrightarrow{\text{switch}}\ldots \xrightarrow{\text{switch}}\]
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At rôle $\hat{r}_k$
- queries through $\mathcal{O}_{\hat{r}_k} \subseteq \mathcal{O}_I$
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Is the identity of an anonymous $x$ hidden w.r.t. $V_{\hat{r}_1}, \ldots, V_{\hat{r}_k}$?

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  $$C^\mathcal{I} \subseteq D^\mathcal{I} \text{ for all models } \mathcal{I} \text{ of } \mathcal{O}_{\hat{r}}.$$  

- Let $q$ be a conjunctive query has $n > 0$ answer variables $\vec{x}$. The answer for $q$ w.r.t. a rôle $\hat{r}$ is a set of tuples of individuals $\vec{t} \in (N_I)^n$, where each $\vec{t}$ replaces $\vec{x}$ and
  
  $$\mathcal{I} \models q(\vec{t}) \text{ for all models } \mathcal{I} \text{ of } \mathcal{O}_{\hat{r}}.$$
Views

Given an ontology $\mathcal{O}_I$

At rôle $\hat{r}_1$
- queries through $\mathcal{O}_{\hat{r}_1} \subseteq \mathcal{O}_I$ \(\xrightarrow{\text{switch}}\) $\ldots$ \(\xrightarrow{\text{switch}}\)
- obtains View $V_{\hat{r}_1}$

At rôle $\hat{r}_k$
- queries through $\mathcal{O}_{\hat{r}_k} \subseteq \mathcal{O}_I$
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Is the identity of an anonymous $x$ hidden w.r.t. $V_{\hat{r}_1}, \ldots, V_{\hat{r}_k}$?

- Let $q = C \sqsubseteq D$ be a subsumption query. The answer for $q$ w.r.t. a rôle $\hat{r}$ is \{true\} if
  \[ C^\mathcal{I} \subseteq D^\mathcal{I} \] for all models $\mathcal{I}$ of $\mathcal{O}_\hat{r}$.

- Let $q$ be a conjunctive query has $n > 0$ answer variables $\vec{x}$.
  The answer for $q$ w.r.t. a rôle $\hat{r}$ is a set of tuples of individuals $\vec{t} \in (\mathcal{N}_I)^n$,
  where each $\vec{t}$ replaces $\vec{x}$ and
  \[ \mathcal{I} \models q(\vec{t}) \] for all models $\mathcal{I}$ of $\mathcal{O}_\hat{r}$.

- A view $V_{\hat{r}}$ is a finite set of pairs of query and answers $\langle q, \text{ans}(q, \hat{r}) \rangle$
The Identity Problem

Is the identity of an anonymous x hidden w.r.t. $V_{\hat{r}_1}, \ldots, V_{\hat{r}_k}$?
The Identity Problem

Is the identity of an anonymous x hidden w.r.t. $V_{\hat{r}_1}, \ldots, V_{\hat{r}_k}$?

**Identity Problem** ($\mathcal{O} \models a \equiv b$)

Given two individuals $a, b$ and an ontology $\mathcal{O}$, check whether

$$a^\mathcal{I} = b^\mathcal{I} \text{ for all models } \mathcal{I} \text{ of } \mathcal{O}$$

---

The Identity Problem

Is the identity of an anonymous $x$ hidden w.r.t. $V_{\hat{r}_1}, \ldots, V_{\hat{r}_k}$?

- **Identity Problem** ($\mathcal{O} \models a \equiv b$)
  Given two individuals $a, b$ and an ontology $\mathcal{O}$, check whether
  $$a^\mathcal{I} = b^\mathcal{I}$$
  for all models $\mathcal{I}$ of $\mathcal{O}$

- **Identity to Instance**: Given two individuals $a, b$, and an ontology $\mathcal{O}$, it holds
  $$\mathcal{O} \models a \equiv b \iff (\mathcal{O} \cup A(a)) \models A(b),$$
  where $A$ is fresh

---

The Identity Problem

Is the identity of an anonymous x hidden w.r.t. $V_{\hat{r}_1}, \ldots, V_{\hat{r}_k}$?

- **Identity Problem** ($\mathcal{O} \models a \doteq b$)
  
  Given two individuals $a, b$ and an ontology $\mathcal{O}$, check whether
  
  $$a^\mathcal{I} = b^\mathcal{I} \text{ for all models } \mathcal{I} \text{ of } \mathcal{O}$$

- **Identity to Instance**: Given two individuals $a, b$, and an ontology $\mathcal{O}$, it holds
  
  $$\mathcal{O} \models a \doteq b \text{ iff } (\mathcal{O} \cup A(a)) \models A(b), \text{ where } A \text{ is fresh}$$

- Only make sense for $\mathcal{O}$ formulated in a **DL with equality power** (with **nominals**, **number restrictions**, or **functional dependencies**)

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Is the identity of an anonymous hidden w.r.t. $V_{\hat{r}_1}, \ldots, V_{\hat{r}_k}$?
Hidden Identity

Is the identity of an anonymous $x$ hidden w.r.t. $V_{\hat{r}_1}, \ldots, V_{\hat{r}_k}$?

- $N_I = N_{KI} \cup N_{AI}$, sets of known and anonymous individuals, respectively
Is the identity of an anonymous \( x \) hidden w.r.t. \( V_{\hat{r}_1}, \ldots, V_{\hat{r}_k} \)?

- \( N_I = N_{KI} \cup N_{AI} \), sets of known and anonymous individuals, respectively
- The identity of \( x \in N_{AI} \) w.r.t. \( \mathcal{D} \) is

\[
\text{idn}(x, \mathcal{D}) = \{ a \in N_{KI} \mid \mathcal{D} \models x = a \}
\]
Is the identity of an anonymous $x$ hidden w.r.t. $V_{\hat{r}_1}, \ldots, V_{\hat{r}_k}$?

- $N_I = N_{KI} \cup N_{AI}$, sets of known and anonymous individuals, respectively
- The identity of $x \in N_{AI}$ w.r.t. $\mathcal{D}$ is
  
  $$idn(x, \mathcal{D}) = \{a \in N_{KI} \mid \mathcal{D} \models x = a\}$$

- The identity of $x \in N_{AI}$ is hidden w.r.t. $\mathcal{D}$ iff $idn(x, \mathcal{D}) = \emptyset$. 
How to solve the View-based Identity Problem?

- Construct an ontology that is compatible with all views $V_{r_1}, \ldots, V_{r_k}$.
How to solve the View-based Identity Problem?

- Construct an ontology that is compatible with all views $V_{\hat{r}_1}, \ldots, V_{\hat{r}_k}$.
- However, there are still variables $\vec{y}$ in some queries $q$ in some $V_{\hat{r}_1}, \ldots, V_{\hat{r}_k}$.
How to solve the View-based Identity Problem?

- Construct an ontology that is compatible with all views \( V_{\hat{r}_1}, \ldots, V_{\hat{r}_k} \).
- However, there are still variables \( \vec{y} \) in some queries \( q \) in some \( V_{\hat{r}_1}, \ldots, V_{\hat{r}_k} \).
- We need to remove those \( \vec{y} \).

**Ground Query**

Given an interpretation \( \mathcal{I} \), a CQ \( q \), and a tuple \( \vec{t} \in (N_r)^n \) such that \( \mathcal{I} \models q(\vec{t}) \), a ground query \( \hat{q} \) is:

- \( \text{conj}(\vec{t}, \vec{u}) \), where \( \vec{u} \) is a tuple of individuals.
- obtained from \( q \) by replacing all variables in \( \vec{y} \) with fresh individuals \( a_y \) over \( \vec{u} \).
How to solve the View-based Identity Problem?

- Construct an ontology that is compatible with all views $V_{\hat{r}_1}, \ldots, V_{\hat{r}_k}$.
- However, there are still variables $\vec{y}$ in some queries $q$ in some $V_{\hat{r}_1}, \ldots, V_{\hat{r}_k}$.
- We need to remove those $\vec{y}$.

Ground Query

Given an interpretation $\mathcal{I}$, a CQ $q$, and a tuple $\vec{t} \in (N_I)^n$ such that $\mathcal{I} \models q(\vec{t})$, a ground query $\hat{q}$ is:

- $\text{conj}(\vec{t}, \vec{u})$, where $\vec{u}$ is a tuple of individuals.
- obtained from $q$ by replacing all variables in $\vec{y}$ with fresh individuals $a_y$ over $\vec{u}$.

Canonical Ontology

The canonical ontology $O_V$ of $V_{\hat{r}_1}, \ldots, V_{\hat{r}_k}$ is defined as $O_V := (\mathcal{T}_V, \mathcal{A}_V)$ where

\[
\mathcal{T}_V := \{ C \sqsubseteq D \mid \langle C \sqsubseteq D, \{ \text{true} \} \rangle \in V_{\hat{r}_i}, \text{ for some } i, 1 \leq i \leq k \} \\
\mathcal{A}_V := \{ A(a) \mid \langle q, \vec{t} \rangle \in V_{r_i} \land A(a) \text{ is a conjunct in } \hat{q}, \text{ for some } i, 1 \leq i \leq k \} \cup \\
\{ r(a, b) \mid \langle q, \vec{t} \rangle \in V_{r_i} \land r(a, b) \text{ is a conjunct in } \hat{q}, \text{ for some } i, 1 \leq i \leq k \}.
\]
How to solve the View-based Identity Problem? \(^2\)

**Theorem**

The identity of \(x \in N_{AI}\) is **hidden** w.r.t. \(V_{\hat{r}_1}, \ldots, V_{\hat{r}_k}\) iff \(\text{idn}(x, O_V) = \emptyset\).

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The identity of $x \in N_{AI}$ is hidden w.r.t. $V\hat{r}_1, \ldots, V\hat{r}_k$ iff $idn(x, O_V) = \emptyset$.

The view-based identity problem in the DL $\mathcal{L}$ can be solved in
- PTime if $\mathcal{L}$ is $ELO$,
- ExpTime if $\mathcal{L} \in \{ALCO, ALCQ\}$,
- NExpTime if $\mathcal{L}$ is $ALCOIQ$.

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Problem 2: $k$-Anonymity

- Avoiding someone to know if the identity of $x$ belongs to \{a\}.
- Avoiding someone to know if the identity of $x$ belongs to \{a_1, \ldots, a_k\} → (Sweeney, 2002).
- Such a formal protection model that is already well-investigated in DBs.
Problem 2: \( k \)-Anonymity

- Avoiding someone to know if the identity of \( x \) belongs to \( \{a\} \).
- Avoiding someone to know if the identity of \( x \) belongs to \( \{a_1, \ldots, a_k\} \) → (Sweeney, 2002).
- Such a formal protection model that is already well-investigated in DBs.

The Non \( k \)-Anonymity Problem

Let \( \mathcal{O} \) be an ontology, \( x \in N_{AI} \), and \( a_1, \ldots, a_k \in N_{KI} \).
\( x \) is not in \( k \)-Anonymity iff for all models \( \mathcal{I} \) of \( \mathcal{O} \),

\[
x^\mathcal{I} \in \{a_1^\mathcal{I}, \ldots, a_k^\mathcal{I}\}
\]
How to solve the $k$-Anonymity Problem?

The identity problem \text{reduced} \rightarrow \text{the instance problem} \text{reduced} \rightarrow \text{the non } k\text{-anonymity problem}

Non $k$-Anonymity to Instance

Let $\mathcal{O}$ be formulated in a DL $\mathcal{L}$ with equality power, $x \in N_{AI}$, $a_1, \ldots, a_k \in N_{KI}$. It holds that for all models $\mathcal{I}$ of $\mathcal{O}$,

$$x^\mathcal{I} \in \{a_1^\mathcal{I}, \ldots, a_k^\mathcal{I}\} \text{ iff } \mathcal{O}' \models A(x),$$

where $\mathcal{O}' := \mathcal{O} \cup \{A(a_i) \mid 1 \leq i \leq k\}$ and $A$ is fresh.
How to solve the $k$-Anonymity Problem?

the identity problem $\rightarrow$ the instance problem $\rightarrow$ the non $k$-anonymity problem

reduced (only for convex DLs)

Non $k$-Anonymity to Identity

Let $\mathcal{O}$ be formulated in $\mathcal{L} \in \{\mathcal{ELO}, DL-Lite_A, CFD_{nc}\}$

If $x \in N_{AI}$, $a_1, \ldots, a_k \in N_{KI}$, then

for all models $\mathcal{I}$ of $\mathcal{O}$, $x^\mathcal{I} \in \{a_1^\mathcal{I}, \ldots, a_k^\mathcal{I}\}$ iff

for all models $\mathcal{I}$ of $\mathcal{O}$, $x^\mathcal{I} = a_i^\mathcal{I}$ for some $1 \leq i \leq k$. 
How to solve the $k$-Anonymity Problem?

- the identity problem
- the instance problem
- the non $k$-anonymity problem

reduced (only for convex DLs)

Complexity of the $k$-Anonymity Problem

- PTime if $\mathcal{L} \in \{\mathcal{ELo}, \mathcal{DL-Lite}_A, \mathcal{CFD}_{nc}\}$.
- ExpTime complete if $\mathcal{L} \in \{\mathcal{ALCO}, \mathcal{ALCQ}\}$.
- NExpTime-complete if $\mathcal{L}$ is $\mathcal{ALCQI}$.
How to solve the $k$-Anonymity Problem?

- the identity problem
- the instance problem
- the non $k$-anonymity problem

Complexity of the $k$-Anonymity Problem

- PTime if $\mathcal{L} \in \{\mathcal{ELO}, DL-Lite_A, CFD_{nc}\}$.
- ExpTime complete if $\mathcal{L} \in \{\mathcal{ALCO}, ALCQ\}$.
- NExpTime-complete if $\mathcal{L}$ is $\mathcal{ALCOT}$. 

Are the complexities of $k$-anonymity and identity always the same?
Problem 3: Ontology Anonymization

What if \( \alpha = (x \doteq a) \) is not hidden in \( \mathcal{O} \)?

- Assume: 
  - \( \mathcal{O} = \mathcal{O}_s \cup \mathcal{O}_r \) is the disjoint union of a static ontology \( \mathcal{O}_s \) and a refutable ontology \( \mathcal{O}_r \).
  - Only the refutable part may be changed.

Ontology Repair

Let our “secret” \( \alpha \) be of the form

- (Identity) \( x \doteq a \)
- (Instance) \( C(x) \)
- (Concept Relationship) \( C \sqsubseteq D \)

Let \( \text{Con}(\mathcal{O}) := \{ \alpha \mid \mathcal{O} \models \alpha \} \) be the set of all consequences of \( \mathcal{O} \).

Let \( \mathcal{O} \models \alpha \) and \( \mathcal{O}_s \not\models \alpha \). The ontology \( \mathcal{O}' \) is a repair of \( \mathcal{O} \) w.r.t. \( \alpha \) if

\[
\text{Con}(\mathcal{O}_s \cup \mathcal{O}') \subseteq \text{Con}(\mathcal{O}) \setminus \{ \alpha \}.
\]

The repair \( \mathcal{O}' \) is an optimal repair of \( \mathcal{O} \) w.r.t. \( \alpha \) if there is no repair \( \mathcal{O}'' \) of \( \mathcal{O} \) w.r.t. \( \alpha \) such that

\[
\text{Con}(\mathcal{O}_s \cup \mathcal{O}') \subset \text{Con}(\mathcal{O}_s \cup \mathcal{O}'')
\]
Problem 3: Ontology Anonymization

- What if $\alpha = (x = a)$ is not hidden in $\mathcal{O}$?
- **Anonymize** $\mathcal{O}$ to $\mathcal{O}'$ such that $\mathcal{O}' \nvdash \alpha$
Problem 3: Ontology Anonymization

- What if \( \alpha = (x \equiv a) \) is not hidden in \( \mathcal{O} \)?
- **Anonymize** \( \mathcal{O} \) to \( \mathcal{O}' \) such that \( \mathcal{O}' \not\models \alpha \rightarrow \text{“Ontology Repair”}! \)
Problem 3: Ontology Anonymization

What if $\alpha = (x \models a)$ is not hidden in $\mathcal{O}$?

Anonymize $\mathcal{O}$ to $\mathcal{O}'$ such that $\mathcal{O}' \not\models \alpha \rightarrow "$Ontology Repair"$!

Assumption:

- $\mathcal{O} = \mathcal{O}_s \cup \mathcal{O}_r$ is the disjoint union of a static ontology $\mathcal{O}_s$ and a refutable ontology $\mathcal{O}_r$.
- Only the refutable part may be changed
Problem 3: Ontology Anonymization

- What if $\alpha = (x \doteq a)$ is not hidden in $\mathcal{O}$??
- **Anonymize** $\mathcal{O}$ to $\mathcal{O}'$ such that $\mathcal{O}' \not\models \alpha \rightarrow \text{“Ontology Repair”}!$

**Assumption:**
- $\mathcal{O} = \mathcal{O}_s \cup \mathcal{O}_r$ is the disjoint union of a **static ontology** $\mathcal{O}_s$ and a **refutable ontology** $\mathcal{O}_r$.
- Only the refutable part may be changed

### Ontology Repair

- Let us say that our “secret” $\alpha$ is of the form
  (Identity) $x \doteq a$  (Instance) $C(x)$  (Concept Relationship) $C \sqsubseteq D$
Problem 3: Ontology Anonymization

- What if $\alpha = (x \equiv a)$ is not hidden in $\mathcal{O}$?
- **Anonymize** $\mathcal{O}$ to $\mathcal{O}'$ such that $\mathcal{O}' \not\models \alpha \rightarrow \text{"Ontology Repair"}!$
- **Assumption:**
  - $\mathcal{O} = \mathcal{O}_s \cup \mathcal{O}_r$ is the disjoint union of a **static ontology** $\mathcal{O}_s$ and a **refutable ontology** $\mathcal{O}_r$.
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**Ontology Repair**

- Let us say that our “secret” $\alpha$ is of the form
  (Identity) $x \equiv a$      (Instance) $C(x)$      (Concept Relationship) $C \sqsubseteq D$
- Let $Con(\mathcal{O}) := \{\alpha \mid \mathcal{O} \models \alpha\}$ be the set of all **consequences** of $\mathcal{O}$. 

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Problem 3: Ontology Anonymization

- What if \( \alpha = (x \doteq a) \) is not hidden in \( \mathcal{O} \)??
- **Anonymize** \( \mathcal{O} \) to \( \mathcal{O}' \) such that \( \mathcal{O}' \not\models \alpha \rightarrow \text{“Ontology Repair”} \!

**Assumption:**
- \( \mathcal{O} = \mathcal{O}_s \cup \mathcal{O}_r \) is the disjoint union of a **static ontology** \( \mathcal{O}_s \) and a **refutable ontology** \( \mathcal{O}_r \).
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**Ontology Repair**

- Let us say that our “secret” \( \alpha \) is of the form
  - (Identity) \( x \doteq a \)
  - (Instance) \( C(x) \)
  - (Concept Relationship) \( C \sqsubseteq D \)
- Let \( Con(\mathcal{O}) := \{ \alpha \mid \mathcal{O} \models \alpha \} \) be the set of all **consequences** of \( \mathcal{O} \).
- Let \( \mathcal{O} \models \alpha \) and \( \mathcal{O}_s \not\models \alpha \). The ontology \( \mathcal{O}' \) is a **repair** of \( \mathcal{O} \) w.r.t. \( \alpha \) if
  \[
  Con(\mathcal{O}_s \cup \mathcal{O}') \subseteq Con(\mathcal{O}) \setminus \{ \alpha \}
  \]
Problem 3: Ontology Anonymization

- What if $\alpha = (x \doteq a)$ is not hidden in $\mathcal{O}$??
- **Anonymize** $\mathcal{O}$ to $\mathcal{O}'$ such that $\mathcal{O}' \not\models \alpha \rightarrow \text{“Ontology Repair”}!$
- **Assumption:**
  - $\mathcal{O} = \mathcal{O}_s \cup \mathcal{O}_r$ is the disjoint union of a **static ontology** $\mathcal{O}_s$ and a **refutable ontology** $\mathcal{O}_r$.
  - Only the refutable part may be changed

### Ontology Repair

- Let us say that our “secret” $\alpha$ is of the form
  - (Identity) $x \doteq a$
  - (Instance) $C(x)$
  - (Concept Relationship) $C \sqsubseteq D$
- Let $Con(\mathcal{O}) := \{\alpha \mid \mathcal{O} \models \alpha\}$ be the set of all **consequences** of $\mathcal{O}$.
- Let $\mathcal{O} \models \alpha$ and $\mathcal{O}_s \not\models \alpha$. The ontology $\mathcal{O}'$ is a **repair** of $\mathcal{O}$ w.r.t. $\alpha$ if
  $$Con(\mathcal{O}_s \cup \mathcal{O}') \subseteq Con(\mathcal{O}) \setminus \{\alpha\}$$
- The repair $\mathcal{O}'$ is an **optimal repair** of $\mathcal{O}$ w.r.t. $\alpha$ if there is no repair $\mathcal{O}''$ of $\mathcal{O}$ w.r.t. $\alpha$ s.t. $Con(\mathcal{O}_s \cup \mathcal{O}') \subset Con(\mathcal{O}_s \cup \mathcal{O}'')$. 
Let $\mathcal{D} = (\mathcal{T}, \mathcal{A})$ be formulated in $\mathcal{EL}$, where
\[
\mathcal{T} := \{ \exists r. A, \exists r. A \sqsubseteq A \}
\]
\[
\mathcal{A} := \{ A(a) \}
\]
Optimal Repairs Need not Exist!

Let $\mathcal{O} = (\mathcal{T}, \mathcal{A})$ be formulated in $\mathcal{EL}$, where

$$
\mathcal{T} := \{A \sqsubseteq \exists r.A, \exists r.A \sqsubseteq A\}
$$

$$
\mathcal{A} := \{A(a)\}
$$

$\mathcal{O}_s = \mathcal{T}, \mathcal{O}_r = \mathcal{A}$, and the unwanted consequence $\alpha = A(a)$.

Let $\mathcal{O}'$ be a repair. Obviously $\mathcal{O}'$ only contains concept assertions $C(a)$. s.t.

- $C$ does not contain $A$
- $C$ is in the form of $(\exists r.)^n \top (a)$, for $n > 0$. 

Let $\mathcal{O}'' = \{((\exists r.)^n \top (a))\}$ be also a repair.

In addition, $\text{Con}(\mathcal{T} \cup \mathcal{O}') \subset \text{Con}(\mathcal{T} \cup \mathcal{O}'')$ and thus $\mathcal{O}'$ is not optimal.

Since $\mathcal{O}'$ is chosen arbitrarily, this shows there cannot be an optimal repair!
Let $\mathcal{O} = (T, A)$ be formulated in $\mathcal{EL}$, where

$$
T := \{ A \sqsubseteq \exists r. A, \exists r. A \sqsubseteq A \} \\
A := \{ A(a) \} 
$$

$\mathcal{O}_s = T, \mathcal{O}_r = A$, and the unwanted consequence $\alpha = A(a)$.

Let $\mathcal{O}'$ be a repair. Obviously $\mathcal{O}'$ only contains concept assertions $C(a)$. s.t.

- $C$ does not contain $A$
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Since $\mathcal{O}'$ is finite, there is a maximal $n_0$ s.t. $((\exists r.)^{n_0} \top)(a) \in \mathcal{O}'$, but $((\exists r.)^n \top)(a) \not\in \mathcal{O}'$, for all $n > n_0$. 
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Claim: If $\mathcal{O}' = \{(\exists r.)^{n_0}_T(a)\}$, then $(\exists r.)^n_T(a) \notin Con(\mathcal{T} \cup \mathcal{O}')$, for all $n > n_0$. 

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Optimal Repairs Need not Exist!

- Let $\mathcal{O} = (\mathcal{T}, \mathcal{A})$ be formulated in $\mathcal{EL}$, where
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- $\mathcal{O}_s = \mathcal{T}, \mathcal{O}_r = \mathcal{A}$, and the unwanted consequence $\alpha = A(a)$.
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  - $C$ does not contain $A$
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- **Claim:** If $\mathcal{O}' = \{(\exists r.)^{n_0} \top(a)\}$, then $(\exists r.)^n \top(a) \not\in \text{Con}(\mathcal{T} \cup \mathcal{O}')$, for all $n > n_0$.
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Let $\mathcal{O} = (T, A)$ be formulated in $\mathcal{EL}$, where

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1. $C$ does not contain $A$
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Optimal Classical Repair

- The repair $\mathcal{D}'$ is a \textbf{classical repair} of $\mathcal{D}$ w.r.t. $\alpha$ if $\mathcal{D}' \subseteq \mathcal{D}_r$.

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The repair $\mathcal{O}'$ is a **classical repair** of $\mathcal{O}$ w.r.t. $\alpha$ if $\mathcal{O}' \subset \mathcal{O}_r$.

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Optimal classical repairs always exist $\rightarrow$ **Justification** and **Hitting Set**. (Horridge, 2011)

Let $\mathcal{O} \models \alpha$. A **justification** $J$ is a minimal subset of $\mathcal{O}_r$ such that $\mathcal{O}_s \cup J \models \alpha$.

Let $J_1, \ldots, J_k$ be the justifications of $\mathcal{O}$ w.r.t. $\alpha$.
A **hitting set** $\mathcal{H}$ of these justifications is a set of axioms such that $\mathcal{H} \cap J_i \neq \emptyset$ for $i = 1, \ldots, k$. 
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$\mathcal{O}' := \mathcal{O}_r \setminus \mathcal{H}_{\text{min}}$ is an optimal classical repair of $\mathcal{O}$ w.r.t. $\alpha$ such that $\mathcal{O}_s \cup \mathcal{O}' \not\models \alpha$. 

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- Obtaining Classical Repairs $\rightarrow$ removing axioms from $\mathcal{O}$.
- Instead, we want to weaken axioms in $\mathcal{H}_{\text{min}}$!
- Given axioms $\beta, \gamma$, an axiom $\gamma$ is weaker than $\beta$ if $\text{Con}(\{\gamma\}) \subset \text{Con}(\{\beta\})$
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- Obtaining Classical Repairs → **removing axioms** from $\mathcal{O}$.
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Given axioms $\beta, \gamma$, an axiom $\gamma$ is **weaker than** $\beta$ if $\text{Con}\{\gamma\} \subset \text{Con}\{\beta\}$.

**Algorithm 1:**
For each $\beta \in \mathcal{H}_{\text{min}}$ and all $J_1, \ldots, J_k$ containing $\beta$, replace $\beta$ with **exactly one** $\gamma$, where $\gamma \prec \beta$ and
Obtaining Classical Repairs → removing axioms from $\mathcal{D}$.

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  **Construct $\mathcal{O}'$ obtained** from $\mathcal{O}_r$ by removing $\mathcal{H}_{\text{min}}$ and replace each $\beta \in \mathcal{H}_{\text{min}}$ with the weaker $\gamma$. 
Obtaining Classical Repairs → **removing axioms** from $\mathcal{O}$.

Instead, we want to **weaken axioms** in $\mathcal{H}_{\text{min}}$!

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**Construct** $\mathcal{O}'$ **obtained** from $\mathcal{O}_r$ by **removing** $\mathcal{H}_{\text{min}}$ and **replace** each $\beta \in \mathcal{H}_{\text{min}}$ with the weaker $\gamma$.

**Check** whether $\alpha$ is a consequence of $\mathcal{O}_s \cup \mathcal{O}'$. 
Algorithm 1 is still not enough!

- Using Algorithm 1, $\alpha$ still can be a consequence of $\mathcal{O}_s \cup \mathcal{O}'$.

Example

$\mathcal{O}_s = \emptyset$ and $\mathcal{O}_r = (T, A)$

$T := \{B \sqsubseteq C\}$

$A := \{(C \sqcap B)(a)\}$

$\alpha = C(a) (T \cup A)$

$J := \{(C \sqcap B)(a)\}$

$H_{\min} := \{(C \sqcap B)(a)\}$

Take $(C \sqcap B)(a) \in H_{\min}$, weaken it to $B(a)$

It implies $J \{(C \sqcap B)(a)\} \cup \{B(a)\} \not\models \alpha$

But then, $\mathcal{O}' := \mathcal{O} \{ (C \sqcap B)(a)\} \cup \{B(a)\}$

Solution: Just iterate Algorithm 1 until $\mathcal{O}_s \cup \mathcal{O}' \not\models \alpha$.

Does it terminate? Yes,

The iterative algorithm yields an exponential upper bound on the number of iterations.
Using Algorithm 1, $\alpha$ still can be a consequence of $\mathcal{D}_s \cup \mathcal{D}'$.

**Example**

$\mathcal{D}_s = \emptyset$ and $\mathcal{D}_r = (\mathcal{T}, \mathcal{A})$

$\mathcal{T} := \{B \subseteq C\} \quad \mathcal{A} := \{(C \cap B)(a)\} \quad \alpha = C(a) \quad (\mathcal{T} \cup \mathcal{A}) \models \alpha.$
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$\mathcal{D}_s = \emptyset$ and $\mathcal{D}_r = (\mathcal{T}, \mathcal{A})$

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\begin{align*}
\mathcal{T} & := \{B \subseteq C\} & \mathcal{A} & := \{(C \cap B)(a)\} & \alpha & = C(a) & (\mathcal{T} \cup \mathcal{A}) \models \alpha.
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### Example

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\[ \mathcal{T} := \{B \subseteq C\} \quad \mathcal{A} := \{(C \cap B)(a)\} \quad \alpha = (T \cup A) \models \alpha. \]

\[ J := \{(C \cap B)(a)\} \quad \mathcal{H}_{\min} := \{(C \cap B)(a)\} \]

Take $(C \cap B)(a) \in \mathcal{H}_{\min}$, weaken it to $B(a)$
Using Algorithm 1, $\alpha$ still can be a consequence of $\mathcal{D}_s \cup \mathcal{D}'$.

Example

\[ \mathcal{D}_s = \emptyset \text{ and } \mathcal{D}_r = (T, A) \]

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Take \((C \cap B)(a) \in \mathcal{H}_{\text{min}},\) weaken it to \(B(a)\)

It implies \(J \setminus \{(C \cap B)(a)\} \cup \{B(a)\} \not\models \alpha\)
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But then, $\mathcal{D}' := \mathcal{D} \setminus \{(C \cap B)(a)\} \cup \{B(a)\} \models \alpha.$
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**Example**

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**Does it terminate?** Yes,

The iterative algorithm yields an exponential upper bound on the number of iterations.
Weakening Relation

How do we formally weaken the axioms during the iteration?

The binary relation $\succ$ on axioms is a weakening relation if $\beta \succ \gamma$ implies that $\gamma$ is weaker than $\beta$; well-founded if there is no infinite $\succ$-chain $\beta_1 \succ \beta_2 \succ \beta_3 \succ \ldots$; complete if for any axiom $\beta$ that is not a tautology, there is a tautology $\gamma$ such that $\beta \succ \gamma$. A linear (polynomial) weakening relation shows that the iterative algorithm can terminate after a linear (polynomial) number of iterations.

In the context of Description Logics, if $\alpha = C \sqsubseteq D$, then the idea for weakening $\alpha$ is to generalize $D$ or to specialize $C$. If $\alpha = C(a)$, then the idea for weakening $\alpha$ is to generalize $C$. 

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How do we formally weaken the axioms during the iteration?

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A linear (polynomial) weakening relation shows that the iterative algorithm can terminate after a **linear (polynomial) number of iterations**.
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In the context of Description Logics,

- If $\alpha = C \sqsubseteq D$, then the idea for weakening $\alpha$ is to **generalize** $D$ or to **specialize** $C$.
- If $\alpha = C(a)$, then the idea for weakening $\alpha$ is to **generalize** $C$. 
Future Work

- Investigating weakening relations in some DLs
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- To make repairs as gentle as possible, the following problem is encountered: Given a concept $D$, a concept $F$ is an upper neighbor (UN) of $D$ iff
  - $D \sqsubseteq F$
  - there is no $E$ such that $D \sqsubseteq E \sqsubseteq F$

Given $J \mid \alpha$ and $C \sqsubseteq D \in J$, find $D'$ without "searching blindly" s.t.

- $D \sqsubseteq D'$
- $O \cup (J \{ C \sqsubseteq D \}) \cup \{ C \sqsubseteq D' \} \mid = \alpha$
- $D'$ is as specific as possible

Whatever how you repair/anonymize $O$ to $O'$, please show for all $O''$ s.t. $O'' \not\mid = \alpha$, we have $(O' \cup O'') \not\mid = \alpha$. 

Adrian Nuradiansyah  
Talk in Bolzano 2018  
March 8, 2018  
18 / 19
Future Work

- Investigating weakening relations in some DLs

- To make repairs as gentle as possible, the following problem is encountered: Given a concept $D$, a concept $F$ is an upper neighbor (UN) of $D$ iff
  - $D \sqsubseteq F$
  - there is no $E$ such that $D \sqsubseteq E \sqsubseteq F$

- Given $J \models \alpha$ and $C \sqsubseteq D \in J$, find $D'$ without “searching blindly” s.t.
  - $D \sqsubseteq D'$
  - $\mathcal{O}_s \cup (J \setminus \{C \sqsubseteq D\}) \cup \{C \sqsubseteq D'\} \models \alpha$
  - $D'$ is as specific as possible
Future Work

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  - $D'$ is as specific as possible

- Whatever how you repair/anonymize $\mathcal{O}$ to $\mathcal{O}'$, please show
  for all $\mathcal{O}''$ s.t. $\mathcal{O}'' \not\models \alpha$, we have $(\mathcal{O}' \cup \mathcal{O}'') \not\models \alpha$. 
Thank You