Reasoning in Description Logic Ontologies for Identity Management

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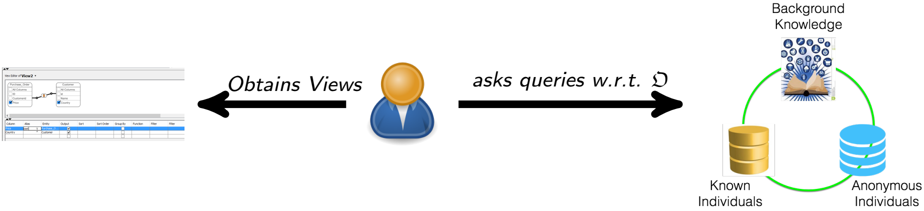
Problem 1: View-based Identity Problem



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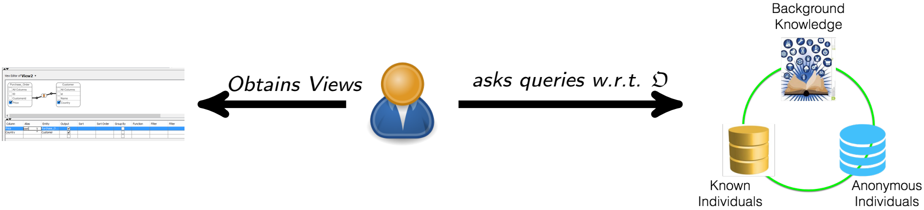

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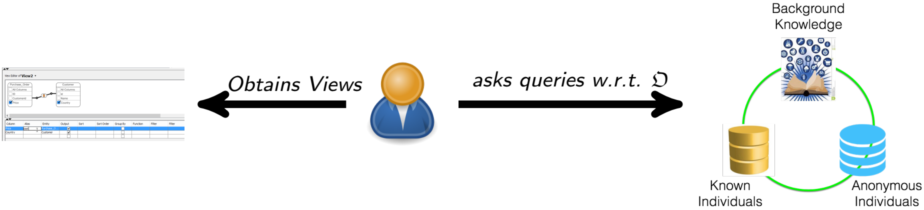

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At rôle $\hat{r}_1$
- queries through $\mathfrak{O}_{\hat{r}_1} \subseteq \mathfrak{O}_I$
- obtains View $V_{\hat{r}_1}$

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At rôle $\hat{r}_k$
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Is the identity of an anonymous x hidden w.r.t. $V_{\hat{r}_1}, \dots, V_{\hat{r}_k}$?

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- An ABox $\mathcal{A}$ is a set of **concept assertions** $C(a)$ and **relationship assertions** $r(a, b)$
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1. **Subsumption Query**: $q = C \sqsubseteq D$, where C and D are DL concepts
2. **Conjunctive Query**: $q(\vec{x}) \leftarrow \exists \vec{y}.conj(\vec{x}, \vec{y})$, where
   - $\vec{x}$ are **answer variables** and $\vec{y}$ are **existentially quantified variables**.
   - $conj(\vec{x}, \vec{y})$ is a **conjunction of atoms** $A(z)$ or $r(z, z')$.

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$$C^{\mathcal{I}} \subseteq D^{\mathcal{I}} \text{ for all models } \mathcal{I} \text{ of } \mathfrak{O}_{\hat{r}}.$$

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The **answer** for q w.r.t. a rôle $\hat{r}$ is a set of **tuples of individuals** $\vec{t} \in (N_I)^n$, where each $\vec{t}$ replaces $\vec{x}$ and

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- A **view** $V_{\hat{r}}$ is a finite **set of pairs of query and answers** $\langle q, ans(q, \hat{r})\rangle$

The Identity Problem [1]

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- Only make sense for $\mathfrak{O}$ formulated in a **DL with equality power** (with **nominals**, **number restrictions**, or **functional dependencies**)

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- The identity of $x \in N_{AI}$ is **hidden** w.r.t. $\mathfrak{O}$ iff $idn(x, \mathfrak{O}) = \emptyset$.

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- We need to remove those $\vec{y}$.

Ground Query

Given an interpretation $\mathcal{I}$, a CQ q, and a tuple $\vec{t} \in (N_I)^n$ such that $\mathcal{I} \models q(\vec{t})$, a **ground query** $\widehat{q}$ is:

- $conj(\vec{t}, \vec{u})$, where $\vec{u}$ is a tuple of individuals.
- obtained from q by **replacing all variables in** $\vec{y}$ with fresh individuals a_y over $\vec{u}$.

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Canonical Ontology

The **canonical ontology** $\mathcal{O}_V$ of $V_{\hat{r}_1}, \ldots, V_{\hat{r}_k}$ is defined as $\mathcal{O}_V := (\mathcal{T}_V, \mathcal{A}_V)$ where

$$\mathcal{T}_V := \{C \sqsubseteq D \mid \langle C \sqsubseteq D, \{\mathtt{true}\}\rangle \in V_{\hat{r}_i} \text{ for some } i, 1 \leq i \leq k\}$$

$$\mathcal{A}_V := \{A(a) \mid \langle q, \vec{t}\rangle \in V_{r_i} \wedge A(a) \text{ is a conjunct in } \widehat{q}, \text{ for some } i, 1 \leq i \leq k\} \cup \{r(a,b) \mid \langle q, \vec{t}\rangle \in V_{r_i} \wedge r(a,b) \text{ is a conjunct in } \widehat{q}, \text{ for some } i, 1 \leq i \leq k\}.$$

How to solve the View-based Identity Problem? [2]

Theorem

The identity of $x \in N_{AI}$ is **hidden** w.r.t. $V_{\hat{r}_1}, \ldots, V_{\hat{r}_k}$ iff $idn(x, \mathcal{O}_V) = \emptyset$.

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Complexity

The view-based identity problem in the DL $\mathcal{L}$ can be solved in

- PTime if $\mathcal{L}$ is $\mathcal{ELO}$,
- ExpTime if $\mathcal{L} \in \{\mathcal{ALCO}, \mathcal{ALCQ}\}$,
- NExpTime if $\mathcal{L}$ is $\mathcal{ALCOIQ}$.

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Problem 2: k-Anonymity

- Avoiding someone to know if the identity of x belongs to $\{a\}$.
- Avoiding someone to know if the identity of x belongs to $\{a_1, \ldots, a_k\}$ $\rightarrow$ (Sweeney, 2002).
- Such a formal protection model that is already well-investigated in DBs.

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The Non k-Anonymity Problem

Let $\mathfrak{O}$ be an ontology, $x \in N_{AI}$, and $a_1, \dots, a_k \in N_{KI}$.
x is **not in k-Anonymity** iff for all models $\mathcal{I}$ of $\mathfrak{O}$,

$$x^{\mathcal{I}} \in \{a_1^{\mathcal{I}}, \dots, a_k^{\mathcal{I}}\}$$

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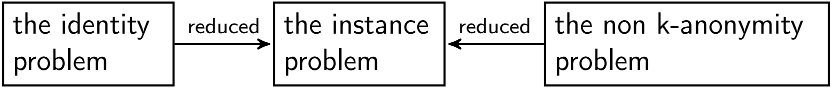

Non k-Anonymity to Instance

Let $\mathfrak{O}$ be formulated in a DL $\mathcal{L}$ with equality power, $x \in N_{AI}$, $a_1, \ldots, a_k \in N_{KI}$.
It holds that for all models $\mathcal{I}$ of $\mathfrak{O}$,

$$x^{\mathcal{I}} \in \{a_1^{\mathcal{I}}, \ldots, a_k^{\mathcal{I}}\} \text{ iff } \mathfrak{O}' \models A(x),$$

where $\mathfrak{O}' := \mathfrak{O} \cup \{A(a_i) \mid 1 \leq i \leq k\}$ and A is fresh.

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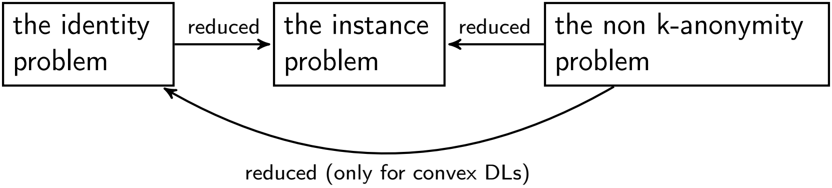

Non k-Anonymity to Identity

Let $\mathfrak{O}$ be formulated in $\mathcal{L} \in \{\mathcal{ELO}, DL\text{-}Lite_A, \mathcal{CFD}_{nc}\}$
If $x \in N_{AI}$, $a_1, \ldots, a_k \in N_{KI}$, then
for all models $\mathcal{I}$ of $\mathfrak{O}$, $x^{\mathcal{I}} \in \{a_1^{\mathcal{I}}, \ldots, a_k^{\mathcal{I}}\}$ iff
for all models $\mathcal{I}$ of $\mathfrak{O}$, $x^{\mathcal{I}} = a_i^{\mathcal{I}}$ for some $1 \leq i \leq k$.

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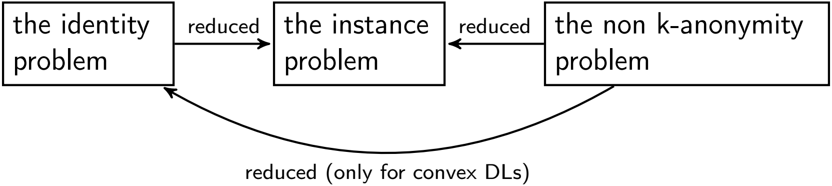

Complexity of the k-Anonymity Problem

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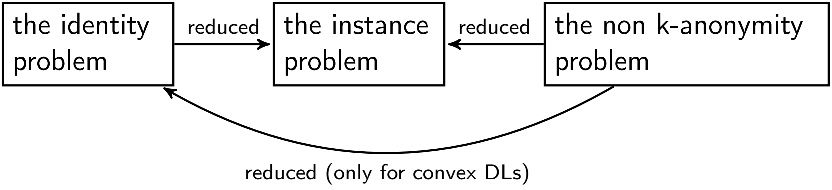

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Are the complexities of k-anonymity and identity always the same?

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- **Assumption:**
  - $\mathfrak{O} = \mathfrak{O}_s \cup \mathfrak{O}_r$ is the disjoint union of a **static ontology** $\mathfrak{O}_s$ and a **refutable ontology** $\mathfrak{O}_r$.
  - Only the refutable part may be changed

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  (Identity) $x \doteq a$ (Instance) $C(x)$ (Concept Relationship) $C \sqsubseteq D$

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- The repair $\mathfrak{O}$' is an **optimal repair** of $\mathfrak{O}$ w.r.t. α if there is no repair $\mathfrak{O}$'' of $\mathfrak{O}$ w.r.t. α s.t. $Con(\mathfrak{O}_s \cup \mathfrak{O}') \subset Con(\mathfrak{O}_s \cup \mathfrak{O}'')$.

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- Let $\mathfrak{O} = (\mathcal{T}, \mathcal{A})$ be formulated in $\mathcal{EL}$, where

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- Since $\mathfrak{O}'$ is chosen arbitrarily, this shows **there cannot be an optimal repair**!

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- The repair $\mathfrak{O}'$ is a **classical repair** of $\mathfrak{O}$ w.r.t. α if $\mathfrak{O}' \subset \mathfrak{O}_r$.
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- Optimal classical repairs always exist $\rightarrow$ **Justification** and **Hitting Set**. (Horridge, 2011)
- Let $\mathfrak{O} \models \alpha$. A **justification** J is a minimal subset of $\mathfrak{O}_r$ such that $\mathfrak{O}_s \cup J \models \alpha$.
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- Obtaining Classical Repairs $\rightarrow$ **removing axioms** from $\mathfrak{O}$.
- Instead, we want to **weaken axioms** in $\mathcal{H}_{\mathsf{min}}$!
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  Check whether α is a consequence of $\mathfrak{O}_s \cup \mathfrak{O}'$.

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- Does it terminate? Yes,
  The iterative algorithm yields **an exponential upper bound** on the number of iterations.

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  - $D \sqsubset D'$
  - $\mathfrak{O}_s \cup (J \setminus \{C \sqsubseteq D\}) \cup \{C \sqsubseteq D'\} \models \alpha$
  - D' is as specific as possible
- Whatever how you repair/anonymize $\mathfrak{O}$ to $\mathfrak{O}'$, please show

$$\text{for all } \mathfrak{O}'' \text{ s.t. } \mathfrak{O}'' \not\models \alpha, \text{ we have } (\mathfrak{O}' \cup \mathfrak{O}'') \not\models \alpha.$$

Thank You

ROSI