# Mixing Description Logics in Privacy-Preserving Ontology Publishing

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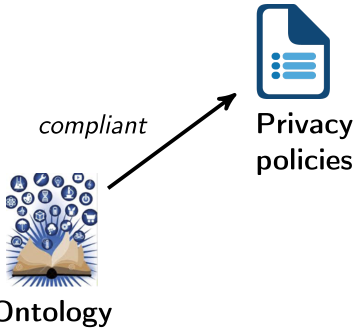
September 25, 2019

# Privacy-Preserving Ontology Publishing (PPOP)



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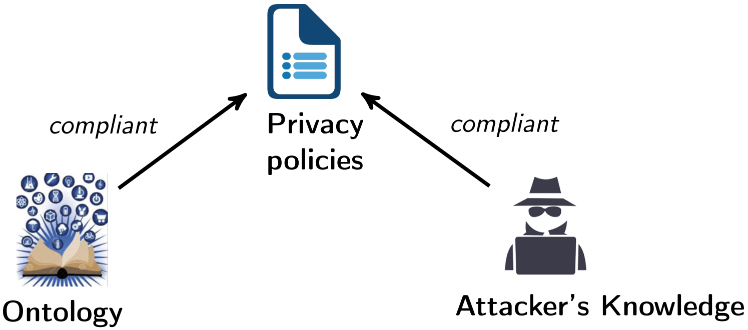

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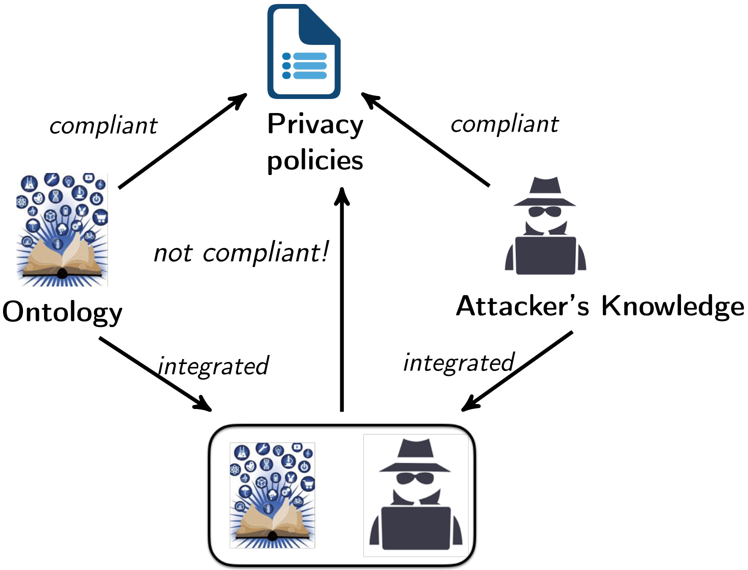

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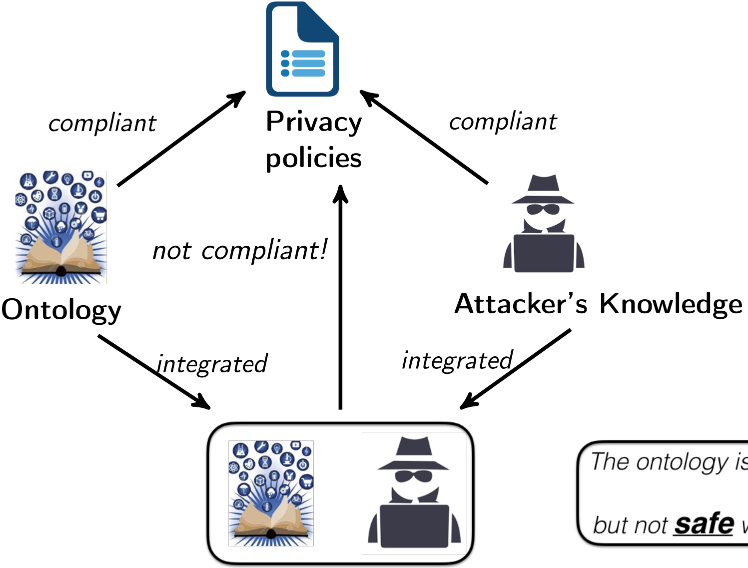


*The ontology is **compliant**,*

*but not **safe** w.r.t. policy!!*

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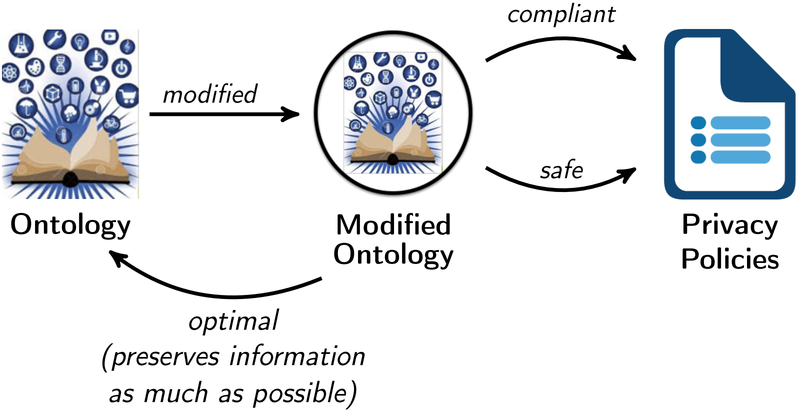

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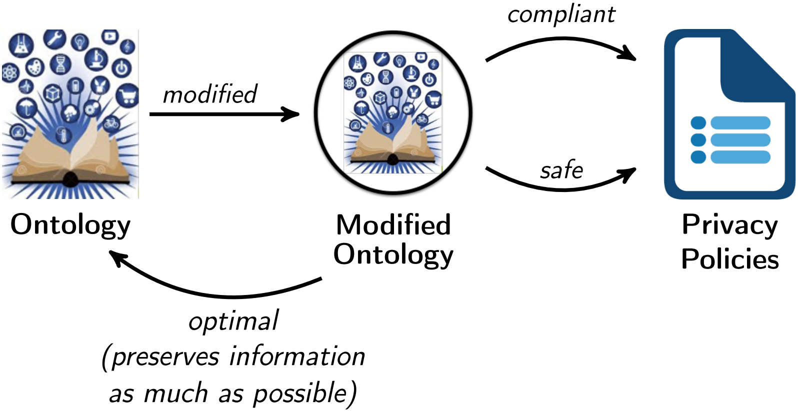


***Assumption:*** *Ontologies are formulated in Description Logics (DLs).*

*What are DLs?*

# Description Logics

- The **logical underpinning** of **Web Ontology Language (OWL)**
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- Decidable fragments of First Order Logics

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- The **basic building blocks** are:
  - $N_C$: set of **concept names** $A$: *Female*, *Doctor*, *Patient*, ...
  - $N_R$: set of **role names** $r$: *seenBy*, *suffer*, *hasSymptom*, ...
  - $N_I$: set of **individual names** $a$: *LINDA*, *CANCER* ...

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  - $N_I$: set of **individual names** $a$: *LINDA*, *CANCER* ...
- The formal semantics is introduced by means of an **interpretation** $(\mathcal{I} = \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$
  - $\Delta^{\mathcal{I}}$ : Non-empty domain elements
  - $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
  - $r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
  - $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$
- Using $N_C$, $N_R$, and $N_I$ as well as necessary constructors, the notion of **DL concepts** $C, D, E$ are built.

# Description Logic Ontologies

- A DL ontology $\mathfrak{O}$ consists of a **TBox** $\mathcal{T}$ and an **ABox** $\mathcal{A}$
- A TBox $\mathcal{T}$ is a set of **General Concept Inclusions (GCIs)** $C \sqsubseteq D$
  $\rightarrow$ hierarchical relationship between concepts
- An ABox $\mathcal{A}$ is a set of **concept assertions** $C(a)$ and **role assertions** $r(a, b)$
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- A DL **Instance Store** $\mathfrak{O}'$ is a DL ontology without role assertions
- A main reasoning task in DLs $\Rightarrow$ Deciding **subsumption** between concepts
- A concept $C$ is **subsumed** by a concept $D$, denoted by $C \sqsubseteq D$, iff $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ for all interpretations $\mathcal{I}$.

# Description Logic $\mathcal{FLE}$ and Its Fragments

- $\mathcal{FLE}$ concepts $C ::= \top$ (*top*) $\mid A \mid C \sqcap C$ (*conjunction*) $\mid$
  $\exists r.C$ (**existential restriction**) $\mid \forall r.C$ (**universal restriction**)

- **Semantics** of some $\mathcal{FLE}$ concepts:
  - $(\exists r.C)^{\mathcal{I}} = \{d \mid \text{there is } e \in \Delta^{\mathcal{I}} \text{ such that } (d,e) \in r^{\mathcal{I}} \wedge e \in C^{\mathcal{I}}\}$
  - $(\forall r.C)^{\mathcal{I}} = \{d \mid \text{for all } e \in \Delta^{\mathcal{I}} \text{ if } (d,e) \in r^{\mathcal{I}}, \text{ then } e \in C^{\mathcal{I}}\}$

- **Fragments** of $\mathcal{FLE}$:
  - the DL $\mathcal{EL}$ (excluding value restrictions)
  - the DL $\mathcal{FL}_0$ (excluding existential restrictions)

$\mathcal{EL}$ Instance Stores
without TBox

# Problem Setting: PPOP for $\mathcal{EL}$ Instance Stores

$\mathcal{EL}$ Instance Stores
without TBox

$C_1(a), C_2(a)$ implies $(C_1 \sqcap C_2)(a)$

only one concept assertion
speaking about one individual

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Published
Information

**(an $\mathcal{EL}$ Concept $C$)**

Attacker's
Knowledge

**(an $\mathcal{EL}$ / $\mathcal{FL}_0$ / $\mathcal{FLE}$ Concept $E$)**

Privacy Policy

**(a finite set of $\mathcal{EL}$ concepts)** $\{D_1, \ldots, D_p\}$

# Formalizing Sensitive Information in $\mathcal{EL}$ Instance Stores

- Given an $\mathcal{EL}$ concept $C$ (published information) and an $\mathcal{EL}$ policy $\mathcal{P}$
- Given a quantifier symbol $Q \in \{\exists, \forall, \forall\exists\}$ and a DL $\mathcal{L}_\exists = \mathcal{EL}, \mathcal{L}_\forall = \mathcal{FL}_0, \mathcal{L}_{\forall\exists} = \mathcal{FLE}$

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## Compliance, Safety, Optimality

1. the $\mathcal{L}_Q$ concept $C'$ is **compliant with** $\mathcal{P}$ if $C' \not\sqsubseteq D_i$ for all $i = 1, \ldots, p$,

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   - $Q$-**safe for** $\mathcal{P}$ if for all $\mathcal{L}_Q$ concepts $E$ (attackers' knowledge) that are compliant with $\mathcal{P}$, $C' \sqcap E$ is also compliant with $\mathcal{P}$,

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   - a $Q$-**safe generalization** of $C$ for $\mathcal{P}$ if $C \sqsubseteq C'$ and $C'$ is $Q$-safe for $\mathcal{P}$,

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   - a **$Q$-safe generalization** of $C$ for $\mathcal{P}$ if $C \sqsubseteq C'$ and $C'$ is $Q$-safe for $\mathcal{P}$,
   - an **optimal $Q$-safe generalization** of $C$ for $\mathcal{P}$ if
     - $C'$ is a $Q$-safe generalization of $C$ for $\mathcal{P}$ and
     - there is no $Q$-safe generalization $C''$ of $C$ for $\mathcal{P}$ s.t. $C'' \sqsubset C'$.

# An Illustration on Privacy Attacks in $\mathcal{EL}$ Instance Stores

**Privacy Policy** $\mathcal{P} = \{D\}$ about *LINDA*

$$D = Patient \sqcap \exists seenBy.(Doctor \sqcap \exists worksIn.Oncology)$$

Original **Published Information** $C$ about *LINDA*

$$C = Patient \sqcap Female \sqcap \exists seenBy.(Doctor \sqcap Male \sqcap \exists worksIn.Oncology)$$

Note $C$ is not **compliant with** and $Q$-**safe** for $D$ for $Q \in \{\exists, \forall, \forall\exists\}$

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**Modification**

$$C_1 = Female \sqcap \exists seenBy.(Doctor \sqcap Male \sqcap \exists worksIn.Oncology)$$

Note $C \sqsubseteq C_1$ and $C_1$ **complies with** $D$

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$\exists$-**Attacker is Coming!**

$$C_1 = Female \sqcap \exists seenBy.(Doctor \sqcap Male \sqcap \exists worksIn.Oncology)$$

He knows $Patient(LINDA)$

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Note $C$ is not **compliant with** and $Q$-**safe** for $D$ for $Q \in \{\exists, \forall, \forall\exists\}$

**Linked and Revealed!**

$$C_1 = Female \sqcap \exists seenBy.(Doctor \sqcap Male \sqcap \exists worksIn.Oncology) \sqcap \boldsymbol{Patient}$$

Note $D(LINDA)$ is **revealed**

# An Illustration on Privacy Attacks in $\mathcal{EL}$ Instance Stores

**Privacy Policy** $\mathcal{P} = \{D\}$ about *LINDA*

$$D = \mathit{Patient} \sqcap \exists \mathit{seenBy}.(\mathit{Doctor} \sqcap \exists \mathit{worksIn}.\mathit{Oncology})$$

Original **Published Information** $C$ about *LINDA*

$$\begin{aligned} C = {} & \mathit{Patient} \sqcap \mathit{Female} \\ & \sqcap \exists \mathit{seenBy}.(\mathit{Doctor} \sqcap \mathit{Male} \sqcap \exists \mathit{worksIn}.\mathit{Oncology}) \end{aligned}$$

Note $C$ is not **compliant with** and $Q$-**safe** for $D$ for $Q \in \{\exists, \forall, \forall\exists\}$

**Modification**

$$\begin{aligned} C_2 = {} & \mathit{Female} \sqcap \exists \mathit{seenBy}.(\mathit{Doctor} \sqcap \mathit{Male} \sqcap \exists \mathit{worksIn}.\top) \\ & \sqcap \exists \mathit{seenBy}.(\mathit{Male} \sqcap \mathit{worksIn}.\mathit{Oncology}) \end{aligned}$$

$C_2$ is the (unique) **optimal** $\exists$-**safe generalization** for $D$

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Note $C$ is not **compliant with** and $Q$**-safe** for $D$ for $Q \in \{\exists, \forall, \forall\exists\}$

**$\forall$-Attacker is Coming!**

$$C_2 = \mathit{Female} \sqcap \exists \mathit{seenBy}.(\mathit{Doctor} \sqcap \mathit{Male} \sqcap \exists \mathit{worksIn}.\top) \sqcap \exists \mathit{seenBy}.(\mathit{Male} \sqcap \mathit{worksIn}.\mathit{Oncology})$$

He knows $(\mathit{Patient} \sqcap \forall \mathit{seenBy}.\forall \mathit{worksIn}.\mathit{Oncology})(\mathit{LINDA})$

# An Illustration on Privacy Attacks in $\mathcal{EL}$ Instance Stores

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$D(\mathit{LINDA})$ is revealed again

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Note $C$ is not **compliant with** and $Q$-**safe** for $D$ for $Q \in \{\exists, \forall, \forall\exists\}$

**Modification**

$$C_3 = \mathit{Female} \sqcap \mathit{Patient} \sqcap \exists \mathit{seenBy}.(\mathit{Doctor} \sqcap \mathit{Male})$$

Note $C_3$ is an **optimal** $\forall$-**safe generalization** for $D$

# An Illustration on Privacy Attacks in $\mathcal{EL}$ Instance Stores

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Note $C$ is not **compliant with** and $Q$-**safe** for $D$ for $Q \in \{\exists, \forall, \forall\exists\}$

$\forall\exists$-**Attacker is Coming!**

$$C_3 = Female \sqcap Patient \sqcap \exists seenBy.(Doctor \sqcap Male)$$

He knows $(\forall seenBy.\exists worksIn.Oncology)(LINDA)$

# An Illustration on Privacy Attacks in $\mathcal{EL}$ Instance Stores

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$$C_3 = Female \sqcap Patient \sqcap \exists seenBy.(Doctor \sqcap Male) \sqcap \forall \mathbf{seenBy}.\exists \mathbf{worksIn.Oncology}$$

$D(LINDA)$ is **revealed again**

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Note $C$ is not **compliant with** and $Q$-**safe** for $D$ for $Q \in \{\exists, \forall, \forall\exists\}$

**Modification**

$$C_4 = Female$$

Note $C_4$ is the **optimal** $\forall\exists$-**safe generalization** for $D$!

# Our Decision and Computational Problems

Given $Q \in \{\forall, \forall\exists\}$, a published information ($\mathcal{EL}$ concept) $C$, an $\mathcal{EL}$ policy $\mathcal{P}$.

## Decision Problems

- **$Q$-Safety**:
  *Is an $\mathcal{EL}$ concept $C_1$ $Q$-safe for a policy $\mathcal{P}$?*
- **$Q$-Optimality**:
  *Is an $\mathcal{EL}$ concept $C_1$ an optimal $Q$-safe generalization of $C$ for $\mathcal{P}$?*

## Computational Problem

*Find an $\mathcal{EL}$ concept $C_1$ s.t $C_1$ is an optimal $Q$-safe generalization of $C$ for $\mathcal{P}$!*

**compliance**, **$\exists$-safety** and **$\exists$-optimality** have been investigated by (Baader, Kriegel, Nuradiansyah in JELIA 2019)

# Complexity Results

| **Decision Problems** | $\mathbf{Q} = \exists$ | $\mathbf{Q} = \forall$ | $\mathbf{Q} = \forall\exists$ |
|---|---|---|---|
| $Q$-safety | PTime* | PTime | PTime |
| $Q$-optimality | coNP* and Dual-hard* | coNP and Dual-hard | PTime |

Table: Complexity results of decision problems on PPOP for $\mathcal{EL}$ instance stores

| **Computational Problems** | $\mathbf{Q} = \exists$ | $\mathbf{Q} = \forall$ | $\mathbf{Q} = \forall\exists$ |
|---|---|---|---|
| Optimal $Q$-safe Generalization(s) | ExpTime* | ExpTime | PTime |

Table: Complexity of computing one/all optimal $Q$-safe generalizations for $\mathcal{P}$

* investigated by (Baader, Kriegel, and Nuradiansyah in JELIA 2019)

# Complexity Results

| **Decision Problems** | $\mathbf{Q} = \forall$ | $\mathbf{Q} = \forall$ | $\mathbf{Q} = \forall\exists$ |
|---|---|---|---|
| $Q$-safety | PTime* | PTime | PTime |
| $Q$-optimality | coNP* and Dual-hard* | coNP and Dual-hard | PTime |

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|---|---|---|---|
| Optimal $Q$-safe Generalization(s) | ExpTime* | **ExpTime** | PTime |

## Reasons:

- Given an $\mathcal{EL}$ concept $D$, $\mathtt{con}(D)$ is the **set of all atoms** ($A$ or $\exists r.D'$) in the top-level conjunction of $D$.
- Computing all **minimal hitting sets** of $\mathtt{con}(D_1), \ldots, \mathtt{con}(D_p)$, where $\mathcal{P} = \{D_1, \ldots, D_p\}$.
- The computation is performed recursively on the **role depth** of the published information $C$

# Complexity Results

| **Decision Problems** | $\mathbf{Q} = \exists$ | $\mathbf{Q} = \forall$ | $\mathbf{Q} = \forall\exists$ |
|---|---|---|---|
| $Q$-safety | PTime$^*$ | PTime | PTime |
| $Q$-optimality | coNP$^*$ and Dual-hard$^*$ | **coNP** and Dual-hard | PTime |

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| Optimal $Q$-safe Generalization(s) | ExpTime$^*$ | ExpTime | PTime |

## Reasons:

- Check if $C_1$ is an $\forall$-safe generalization of $C$ for $\mathcal{P}$
- Check if there is $C_2$ s.t. $C \sqsubseteq C_2 \sqsubset C_1$, where $C_2$ is a not $\forall$-safe generalization of $C$ for $\mathcal{P}$
- There is an NP algorithm to **guess** such concept $C_2$ (Baader, Kriegel, Nuradiansyah in JELIA 2019)

# Complexity Results

| **Decision Problems** | $\mathbf{Q} = \exists$ | $\mathbf{Q} = \forall$ | $\mathbf{Q} = \forall\exists$ |
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| $Q$-safety | PTime* | PTime | PTime |
| $Q$-optimality | coNP* and Dual-hard* | coNP and **Dual-hard** | PTime |

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|---|---|---|---|
| Optimal $Q$-safe Generalization(s) | ExpTime* | ExpTime | PTime |

## Reasons:

- $\forall$-Optimality is coNP-hard? Don't know yet
- There is a polynomial reduction of Dual problem to $\forall$-optimality

  Given two **families of inclusion-comparable sets** $\mathcal{G}$ and $\mathcal{H}$, Dual asks whether $\mathcal{H}$ consists exactly of the minimal hitting sets of $\mathcal{G}$.

# Complexity Results

| **Decision Problems** | **Q** $= \exists$ | **Q** $= \forall$ | **Q** $= \forall\exists$ |
|---|---|---|---|
| $Q$-safety | PTime* | PTime | **PTime** |
| $Q$-optimality | coNP* and Dual-hard* | coNP and Dual-hard | **PTime** |

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**Reasons:**

## $\forall\exists$-Safety and $\forall\exists$-Optimality

$C$ is **$\forall\exists$-safe for $\mathcal{P}$** iff

1. $A \notin \mathsf{con}(C)$ for all concept names $A \in \mathsf{con}(D_1) \cup \ldots \cup \mathsf{con}(D_p)$, and
2. for all existential restrictions $\exists r.D' \in \mathsf{con}(D_1) \cup \ldots \cup \mathsf{con}(D_p)$, there is no concept of the form $\exists r.E \in \mathsf{con}(C)$

# Conclusions and Future Work

**Conclusions:**

- Investigate **PPOP for $\mathcal{EL}$ Instance Stores**
- Considering **attacker's knowledge** to be given by an $\mathcal{FL}_0$ or $\mathcal{FLE}$ concept
- Deciding $Q$-**safety** and $Q$-**optimality**, where $Q \in \{\forall, \forall\exists\}$.
- Computing **optimal** $Q$**-safe generalizations** of $\mathcal{EL}$ concepts for $\mathcal{P}$

**Note:** *the stronger the attacker's knowledge, the more radical we need to change the concept to make it safe*

# Conclusions and Future Work

**Conclusions:**

- Investigate **PPOP for $\mathcal{EL}$ Instance Stores**
- Considering **attacker's knowledge** to be given by an $\mathcal{FL}_0$ or $\mathcal{FLE}$ concept
- Deciding $Q$-**safety** and $Q$-**optimality**, where $Q \in \{\forall, \forall\exists\}$.
- Computing **optimal $Q$-safe generalizations** of $\mathcal{EL}$ concepts for $\mathcal{P}$

**Note:** *the stronger the attacker's knowledge, the more radical we need to change the concept to make it safe*

**Future Work:**

- PPOP in $\mathcal{EL}$ ABoxes, including role assertions (Ongoing!)
- PPOP in $\mathcal{EL}$ Instance Stores w.r.t. (General) TBoxes
- Playing with more different or expressive DLs

# Thank You