Making Repairs in Description Logics More Gentle

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Motivation

- Reasoning in large ontologies $\mathfrak{O}$ may provide **unintended consequences** α
$\Rightarrow$ $\mathfrak{O}$ contains **errors**.
- In privacy setting, some (correct) consequences α should be **hidden from attackers**.
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What we want to do:

- Instead of removing axioms, we propose axiom weakenings.
- Addressed in the context of Description Logic Ontologies

$\mathcal{EL}$ Ontologies

- $\mathcal{EL}$-concepts $C, D ::= \top \mid A \mid C \sqcap D \mid \exists r.C$.
- Inexpressive, but reasoning can be done in **polynomial time**.
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- An **ontology** $\mathfrak{O}$ consists of **TBox** $\mathcal{T}$ and **ABox** $\mathcal{A}$.
- A TBox $\mathcal{T}$ is a finite set of **General Concept Inclusions (GCIs)** $C \sqsubseteq D$
  $\rightarrow$ Background knowledge
- An ABox $\mathcal{A}$ is a finite set of **concept assertions** $C(a)$ and **role assertions** $r(a, b)$
  $\rightarrow$ Knowledge about individuals

Ontology Repair

Assumptions:

- $\mathfrak{O} = \mathfrak{O}_s \cup \mathfrak{O}_r$, where $\mathfrak{O}_s$ is a **static ontology** and $\mathfrak{O}_r$ is a **refutable ontology**.
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Consider: $\mathcal{T} := \{A \sqsubseteq \exists r.A, \exists r.A \sqsubseteq A\}$ $\quad \mathcal{A} := \{A(a)\}$ $\quad \alpha = A(a)$

If $\mathfrak{O}_r := \mathcal{A}$, then an optimal repair must contain $((\exists r.)^n\top)(a)$ for **infinitely many** n

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- Let $\mathfrak{O} \models \alpha$. A **justification** J of $\mathfrak{O}$ w.r.t. α is a minimal subset of $\mathfrak{O}_r$ s.t. $\mathfrak{O}_s \cup J \models \alpha$.
- Let $J_1, \ldots, J_k$ be the justifications of $\mathfrak{O}$ w.r.t. α.
  A **hitting set** $\mathcal{H}$ of $J_1, \ldots, J_k$ is a set of axioms such that $\mathcal{H} \cap J_i \neq \emptyset$

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- $\mathfrak{O}' := \mathfrak{O}_r \setminus \mathcal{H}_{min}$ is an **optimal classical repair** of $\mathfrak{O}$ w.r.t. α such that

$$\mathfrak{O}_s \cup \mathfrak{O}' \not\models \alpha$$

Gentle Repair

- Obtaining Classical Repairs $\rightarrow$ **removing axioms** from $\mathfrak{O}$.
- Instead, we want to **weaken axioms** in $\mathcal{H}_{\text{min}}$!
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Illustration

$$\mathfrak{O}_s := \{\exists owns.(GermanCar \sqcap Diesel) \sqsubseteq \exists gets.Compensations\}$$
$$\mathfrak{O}_r := \{GermanTaxiDriver \sqsubseteq \exists owns.(GermanCar \sqcap Diesel).\}$$

- Every German taxi driver gets compensation w.r.t. $\mathfrak{O}_s \cup \mathfrak{O}_r$.

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- **More gentle**: Weaken β to
  $GermanTaxiDriver \sqsubseteq \exists owns.GermanCar \sqcap \exists owns.Diesel$

Previous Works on Weakening Axioms

- **In (Horridge et.al., 2008) & (Du et.al., 2014),**
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- **In (Lam et.al., 2008)**
  - Using tracing tableau technique from **(Baader & Hollunder, 1995)**
  - To identify which parts of the axioms involved in deriving α
  - Their approach does not always yield a repair

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Theorem (Termination)

- Obtaining gentle repairs **needs Iterations until $\mathfrak{O}_s \cup \mathfrak{O}' \not\models \alpha$.**
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In (Troquard et.al., 2018)

- Weakening axioms via refinement operators **(Lehmann & Hitzler, 2010)**.
- Realized that weakening axioms needs iterations.
- But, no termination proof.

Weakening Relations

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Theorem (Linearity/Polynomiality)

If $\succ$ is linear (polynomial) and complete, then the iterative algorithm stops after a **linear (polynomial) number of iterations.**

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$\succ$ is **effectively finitely branching** if for all axioms β, the set $\{\gamma \mid \beta \succ_1 \gamma\}$ is finite.

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$$\beta \succ_1 \delta_1 \succ_1 \ldots \succ_1 \delta_n \succ_1 \gamma \text{ with}$$

$$\mathfrak{O}_s \cup (J_i \setminus \{\beta\}) \cup \{\gamma\} \not\models \alpha, \text{ but } \mathfrak{O}_s \cup (J_i \setminus \{\beta\}) \cup \{\delta_i\} \models \alpha \ \forall i \in \{1, \ldots, n\}$$

Maximally Strong Weakenings

Theorem (Computing MSWs)

To compute all MSWs, the weakening relation $\succ$ should be **well-founded, complete, one-step generated, and effectively finitely branching.**

Algorithm for Computing MSWs

- There are **only finitely** many γ such that $\beta \succ \gamma$.
- All these γ can be **reached by following** $\succ_1$.
- By **a breadth-first search**, we can compute the set of all γ such that there is a path

$$\beta \succ_1 \delta_1 \succ_1 \ldots \succ_1 \delta_n \succ_1 \gamma \text{ with}$$

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- If this set contains comparable elements, then **remove the weaker ones**.
- The remaining set only consists of all MSWs of β in J_i.

Weakening Axioms in $\mathcal{EL}$

- We define

$$C \sqsubseteq D \succ^s C' \sqsubseteq D' \text{ if } C' \sqsubseteq C,\ D \sqsubseteq D', \text{ and } \{C' \sqsubseteq D'\} \not\models C \sqsubseteq D$$

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- $\succ^s$ is complete, but not well founded.
- Specializing the left-hand side is **not well-founded** in $\mathcal{EL}$.

$$\top \sqsubseteq A \succ \exists r.\top \sqsubseteq A \succ \exists r \exists r.\top \sqsubseteq A \succ \ldots$$

- Generalizing the right-hand side is **well-founded** in $\mathcal{EL}$ **(Baader & Morawska, 2010)**.
- For **assertions** in $\mathcal{A}$:
  - $D(a)$ is weakened by generalizing D
  - $r(a, b)$ is weakened to a tautological axiom

A Weakening Relation $\succ^{sub}$ in $\mathcal{EL}$

- We define

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- It is well-founded, complete, one-step generated, finitely branching, but **not polynomial**.
- $\mid D' \mid$ can be **exponential** in $\mid D \mid$.
- Let $N_n := \{A_1, \ldots, A_{2n}\}$ be a set of $2n$ distinct concept names.

$$\exists r. \bigsqcap N_n \sqsubset \bigsqcap_{X \subseteq N_n \wedge |X| = n} \exists r. \bigsqcap X.$$

- **Exponentially many** $\exists r. \bigsqcap X$ that can be removed.

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Complexity Results

- The Algorithm for computing all maximally strong weakenings in $\mathcal{EL}$ w.r.t. $\succ^{sub}$ has **non-elementary complexity**.
- Deciding if γ is a maximally strong weakening w.r.t. $\succ^{sub}$ is **coNP-hard**.

A Better Fragment $\succ^{syn}$ of $\succ^{sub}$

Syntactic Generalizations

A concept D' is a **syntactic generalization** of D, written $D \sqsubset^{syn} D'$, iff some occurrences of subconcepts $\neq \top$ in D are replaced with $\top$.

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- $\succ^{syn}$ is **linear**, complete, one-step generated, and finitely branching.

- $\mid D \mid > \mid D' \mid$.

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- $\succ^{syn}$ is **linear**, complete, one-step generated, and finitely branching.
- $\mid D \mid > \mid D' \mid$.

Complexity Results

- A single maximally strong weakening w.r.t. $\succ^{syn}$ can be computed in **PTime**.
- All maximally strong weakenings w.r.t. $\succ^{syn}$ can be computed in **ExpTime**.
- Deciding if γ is a maximally strong weakening w.r.t. $\succ^{syn}$ is **coNP-complete**.

Conclusions and Future Work

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- **Framework** for repairing ontologies via weakening axioms rather than deleting
- Introduced **weakening relations** and **maximally strong weakenings**
- Applied the framework in **Description Logic** $\mathcal{EL}$

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Future Work

- More **complexity results** for $\succ^{sub}$
  - Finding better upper bound for deciding whether an axiom is an MSW w.r.t. $\succ^{sub}$
  - Finding a better algorithm to compute MSWs w.r.t. $\succ^{sub}$.
- **Weakening relations** for more expressive logics
  $\Rightarrow \mathcal{ELO}$, $\mathcal{ALC}$, etc.
- **Choosing which axioms** to be weakened and the maximally strong weakenings.

Thank You

ROSI