# Privacy-Preserving Ontology Publishing for $\mathcal{EL}$ Instance Stores

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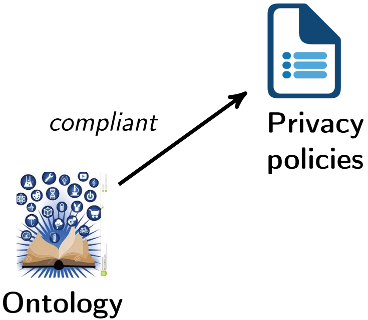

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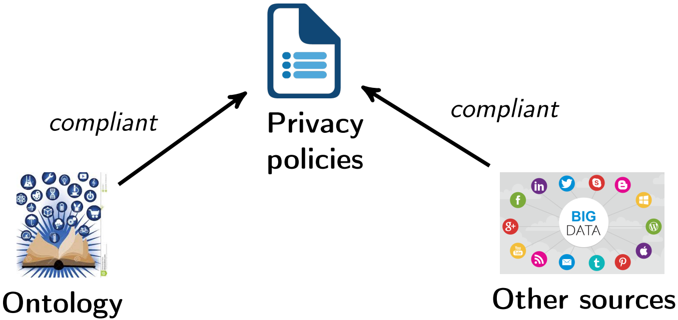

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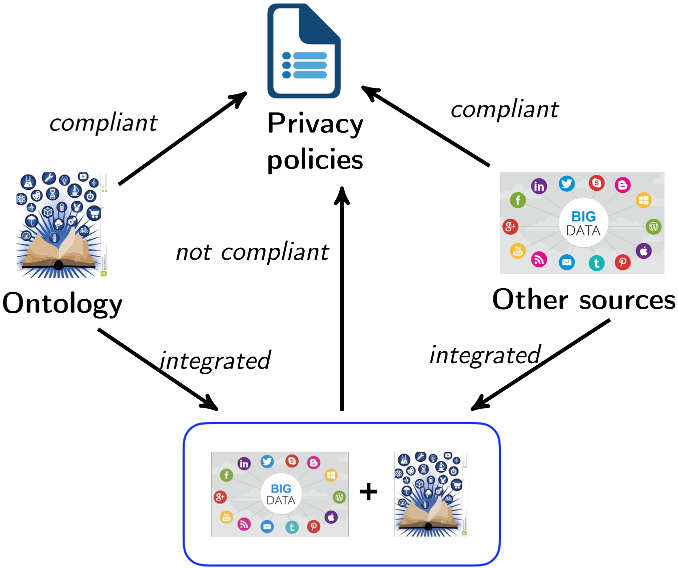

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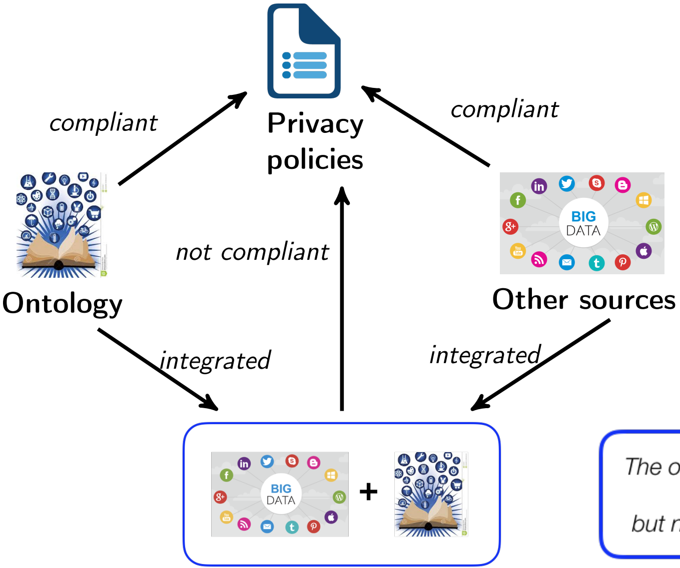


*The ontology is compliant,*

*but not safe w.r.t policies*

# Privacy-Preserving Ontology Publishing

## What people already did:

In **(Cuenca Grau & Kostylev, 2016):**

- Privacy-Preserving Data Publishing
- Information to be published: a relational dataset with (labeled) nulls
- Policy is a conjunctive query.
- Considering three privacy properties when publishing datasets: **policy-compliant, policy-safety, and optimality**.
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## What we want to do:

- **Privacy-Preserving Ontology Publishing (PPOP)**
- Addressed in the context of **Description Logic Ontologies**

# PPOP for $\mathcal{EL}$ instance stores

- **Starting point**: $\mathcal{EL}$ Ontologies with **role-free ABoxes** (**instance stores**) and empty TBoxes.
- An ABox $\mathcal{A}$ is **role-free** if all the axioms $\beta \in \mathcal{A}$ are only in the form of $D(a)$.

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- W.l.o.g., only **one concept assertion** in $\mathcal{A}$ speaks about one individual $C_1(a), C_2(a) \in \mathcal{A}$ implies $(C_1 \sqcap C_2)(a) \in \mathcal{A}$
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- Safe Ontologies $\xrightarrow{\textit{reduced}}$ Safe Concepts
- **Information to be published** for an individual $a$: an $\mathcal{EL}$ concept $C$
- **Policy** is a finite set of $\mathcal{EL}$ concepts $D_1, \ldots, D_p$, such that $D_i \not\equiv \top$ for all $i \in \{1, \ldots, p\}$.

# Compliance, Safety, and Optimality

Given a policy $\mathcal{P} = \{D_1, \ldots, D_p\}$ and an $\mathcal{EL}$ concept $C$, the $\mathcal{EL}$ concept $C'$ is

- **compliant** with $\mathcal{P}$ if $C' \not\sqsubseteq D_i$ for all $i \in \{1, \ldots, p\}$.
- **safe** for $\mathcal{P}$ if $C' \sqcap C''$ is compliant with $\mathcal{P}$ for all $\mathcal{EL}$-concepts $C''$ that are compliant with $\mathcal{P}$.

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- a **$\mathcal{P}$-compliant (safe) generalization** of $C$ if
  - $C \sqsubseteq C'$ and
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- a **$\mathcal{P}$-optimal compliant (safe) generalization** of $C$ if
  - $C'$ is a $\mathcal{P}$-compliant (safe) generalization of $C$, and
  - there is no $\mathcal{P}$-compliant (safe) generalization $C''$ of $C$ s.t. $C'' \sqsubset C'$.

# Illustration on Compliance, Safety, and Optimality

- Consider a **policy** $\mathcal{P} = \{D\}$ specifying what information should be kept "secret" about *linda*

$$D = Patient \sqcap \exists seen\_by.(Doctor \sqcap \exists works\_in.Cardiology)$$

- Assume information $C$ is published about *linda*

$$C = Patient \sqcap Female \sqcap \exists seen\_by.(Doctor \sqcap Male \sqcap \exists works\_in.Cardiology)$$

Note $C$ is not compliant with $D$, i.e., $C \sqsubseteq D$.

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- Generalizing $C$ to yield a compliant concept

$$C_1 = Female \sqcap \exists seen\_by.(Doctor \sqcap Male \sqcap \exists works\_in.Cardiology)$$

  But, $C_1$ is **not safe for** $D$ since if the attacker knows $Patient(linda)$, then $C_1 \sqcap Patient \sqsubseteq D$ is revealed.

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- Let us **make it safe**!

$$C_2 = Female \sqcap \exists seen\_by.(Doctor \sqcap Male \sqcap \exists works\_in.\top)$$

But, $C_2$ is still not optimal since more information than necessary is removed.

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$$\begin{aligned} C_3 = Female \quad & \sqcap \exists seen\_by.(Doctor \sqcap Male \sqcap \exists works\_in.\top) \\ & \sqcap \exists seen\_by.(Male \sqcap \exists works\_in.Cardiology) \end{aligned}$$

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## Complexity for Compliance

- Deciding whether $C'$ is compliant w.r.t. $\mathcal{P}$ is in **PTime.**
- One optimal $\mathcal{P}$-compliant generalization can be **computed in ExpTime**.
- The set of all optimal $\mathcal{P}$-compliant generalizations can be **computed in ExpTime**.

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  $\Rightarrow$ Optimal $\mathcal{P}$-safe generalization is **unique up to equivalence**.
- The $\mathcal{P}$-optimal safe generalization of $C$ can be **computed in ExpTime.**

  $\Rightarrow$ Requiring the computation of $\mathcal{P}$-optimal compliant generalizations.

# Deciding Optimality

- **Deciding** whether $C'$ a $\mathcal{P}$-optimal compliant (safe) generalization of $C$.
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- **Idea**: Design an NP algorithm for deciding non-optimality
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- Only **polynomially many** upper neighbors of $\mathcal{EL}$-concepts and each of them is of **polynomial size** (Kriegel, 2018).
- The next task: **computing lower neighbors!**

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**Lemma**

*$C''$ is a **lower neighbor** of $C'$ w.r.t. $\Sigma$ iff **there is an atom** $At \in LA_\Sigma(C')$ such that $C'' \equiv C' \sqcap At$.*

# Example of Lower Neighbors

## Example

$\Sigma := \{r, A_1, A_2, B_1, B_2, C_1, C_2\}$ and

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To produce exactly the lower neighbors of $C'$ that subsume $C$, let us

- **generate** all $At \in LA_\Sigma(C')$ w.r.t. $\Sigma := sig(C)$, and
- **remove** the ones that do not subsume $C$.

# Generating Lower Neighbors

But $LA_{\Sigma}(C')$ **does not show directly** how appropriate $\exists r.D$ can be found!

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   - If for some $i, 1 \leq i \leq k$, it fails to produce $F_i \in LA_\Sigma(F_i')$, or
   - If $C' \sqsubseteq \exists r.(F_1 \sqcap \ldots \sqcap F_k)$, or
   - If $F_1 \sqcap \ldots \sqcap F_k$ has an upper neighbor $E$ such that $C' \not\sqsubseteq \exists r.E$,

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   - If $F_1 \sqcap \ldots \sqcap F_k$ has an upper neighbor $E$ such that $C' \not\sqsubseteq \exists r.E$,

   then fail. Otherwise, **output** $\exists r.(F_1 \sqcap \ldots \sqcap F_k) \equiv \exists r.D$.

# Complexity for the Optimality Problem

**Theorem**

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**Proposition**

*There is a **polynomial reduction** of* Dual *to the optimality problem for compliance and safety*

# Considering Different Attacker's Knowledge

- What we considered before:
  - Knowledge about individuals
  - Privacy policies
  - Background knowledge of attackers

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- Background Knowledge of Attackers: **$\mathcal{FL}_0$ or $\mathcal{FLE}$ concepts?**
- $\mathcal{FL}_0$ concepts:
$$C, D ::= \top \mid A \mid C \sqcap D \mid \forall r.C$$
- $\mathcal{FLE}$ concepts:
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- Subsumption without general TBoxes:
  - in $\mathcal{FL}_0$: PTime
  - in $\mathcal{FLE}$: NP-complete
- In SNOMED CT, the roles have implicit typing constraints, that may be known to an attacker.

# Extending the Definition of Compliance and Safety

Let $C$ be an $\mathcal{EL}$ concept, $\mathcal{P}$ be an $\mathcal{EL}$ policy, $Q \in \{\forall, \forall\exists\}$, and $\mathcal{L}_\forall = \mathcal{FL}_0, \mathcal{L}_{\forall\exists} = \mathcal{FLE}$.

The $\mathcal{L}_Q$ concept $C'$ is **compliant** with $\mathcal{P}$ if $C' \not\sqsubseteq D$ for all $D \in \mathcal{P}$.

The $\mathcal{EL}$ concept $C'$ is

- **$Q$-safe** for $\mathcal{P}$ if $C' \sqcap C''$ is compliant with $\mathcal{P}$ for all $\mathcal{L}_Q$ concepts $C''$ that are compliant with $\mathcal{P}$.
- a **$Q$-safe generalization** of $C$ for $\mathcal{P}$ if $C \sqsubseteq C'$ and $C'$ is $Q$-safe for $\mathcal{P}$,
- an **optimal $Q$-safe generalization** of $C$ for $\mathcal{P}$ if
  - it is a $Q$-safe generalization of $C$ for $\mathcal{P}$ and
  - there is no $Q$-safe generalization of $C$ for $\mathcal{P}$ such that $C'' \sqsubset C'$.

We now focus on $\forall$-safety and $\forall\exists$-safety

# Illustrations on ∀-Safety and ∀∃-Safety

- Let us consider again

$$D = Patient \sqcap \exists seen\_by.(Doctor \sqcap \exists works\_in.Cardiology)$$

- ...and the published information $C$ about linda

$$C = Patient \sqcap Female \sqcap \exists seen\_by.(Doctor \sqcap Male \sqcap \exists works\_in.Cardiology)$$

Note $C$ is **not compliant** with $D$, i.e., $C \sqsubseteq D$.

- Compute the **optimal safe generalization**

$$\begin{aligned} C_3 = Female \quad & \sqcap \exists seen\_by.(Doctor \sqcap Male \sqcap \exists works\_in.\top) \\ & \sqcap \exists seen\_by.(Male \sqcap \exists works\_in.Cardiology) \end{aligned}$$

But then, if the attacker's knowledge is given by an $\mathcal{FL}_0$ concept $F_1 = \forall seen\_by.\forall works\_in.Cardiology$, then $C_3 \sqcap F_1 \sqsubseteq D$.

# Illustrations on $\forall$-Safety and $\forall\exists$-Safety

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  Note $C$ is **not compliant** with $D$, i.e., $C \sqsubseteq D$.

- Compute an **optimal $\forall$-safe generalization**

$$C_4 = Male \sqcap Patient \sqcap \exists seen\_by.(Doctor \sqcap Female)$$

  However, if the attacker's knowledge is given by an $\mathcal{FLE}$ concept $F_2 = \forall seen\_by.\exists works\_in.Cardiology$, then $C_4 \sqcap F_2 \sqsubseteq D$.

# Illustrations on ∀-Safety and ∀∃-Safety

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$$D = Patient \sqcap \exists seen\_by.(Doctor \sqcap \exists works\_in.Cardiology)$$

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  Note $C$ is **not compliant** with $D$, i.e., $C \sqsubseteq D$.

- Compute an **optimal ∀-safe generalization**

$$C_4 = Male \sqcap Patient \sqcap \exists seen\_by.(Doctor \sqcap Female)$$

  However, if the attacker's knowledge is given by an $\mathcal{FLE}$ concept $F_2 = \forall seen\_by.\exists works\_in.Cardiology$, then $C_4 \sqcap F_2 \sqsubseteq D$.

- Compute the **optimal ∀∃-safe generalization** $C_5 = Male$

# Characterizing ∀-Safety

## ∀-Safety

$C'$ is **∀-safe for $\mathcal{P}$** iff for all $D \in \mathcal{P}$:

1. if $rd(D) = 0$, then $\mathtt{con}(C) \cap \mathtt{con}(D) = \emptyset$.

# Characterizing $\forall$-Safety

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1. if $rd(D) = 0$, then $\mathtt{con}(C) \cap \mathtt{con}(D) = \emptyset$.
2. if $rd(D) > 0$, then there is $\exists r.D' \in \mathtt{con}(D)$ such that
   a. if $rd(D') = 0$, then there is no concept of the form $\exists r.C' \in \mathtt{con}(C)$,
   b. if $rd(D') > 0$, then for all $\exists r.C' \in \mathtt{con}(C)$, $C'$ is $\forall$-safe for $\{D'\}$.

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## Complexity for ∀-Safety

- Deciding whether $C'$ is ∀-safe for $\mathcal{P}$ is in **PTime.**
- One optimal ∀-safe generalization for $\mathcal{P}$ can be **computed in ExpTime**.
- The set of all optimal ∀-safe generalizations for $\mathcal{P}$ can be **computed in ExpTime**.
- ∀-optimality is in **coNP**.

# Characterizing ∀∃-Safety

## ∀∃-Safety

$C$ is **∀∃-safe for $\mathcal{P}$** iff

1. $A \notin \texttt{con}(C)$ for all concept names $A \in \texttt{con}(D_1) \cup \ldots \cup \texttt{con}(D_p)$, and
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## Complexity for ∀-Safety

Given $\mathcal{EL}$ concepts $C, C''$ and a redundancy-free $\mathcal{EL}$ policy $\mathcal{P}$, we

- can decide if $C$ is ∀∃-safe for $\mathcal{P}$,
- can compute the unique optimal ∀∃-safe generalization of $C$ for $\mathcal{P}$, and
- can decide if $C''$ is an optimal ∀∃-safe generalization of $C$ for $\mathcal{P}$

in **polynomial time**

# Conclusions and Future Work

**Conclusions:**

- Define and provide characterizations for **compliance, safety, and optimality** in privacy-preserving ontology publishing for $\mathcal{EL}$ instance stores.
- Computing $\mathcal{P}$**-optimal compliant (safe) generalizations** of $\mathcal{EL}$ concepts.
- Deciding the **optimality problem** via computing **lower neighbors of $\mathcal{EL}$ concepts**.
- Considering **attacker's knowledge** to be given by an $\mathcal{FL}_0$ or $\mathcal{FLE}$ concept.

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**Future Work:**

- PPOP in $\mathcal{EL}$ Instance Stores w.r.t. General TBoxes
- PPOP in $\mathcal{EL}$ ABoxes
- Representing attacker's knowledge with more different DLs

# Thank You

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